PLAN AND PREPARE

Main Ideas

In this chapter students use proportions to identify similar polygons and find the scale factor between two polygons, they use a scale factor to find corresponding lengths in similar polygons, and they use the AA Similarity Postulate, the SSS Similarity Theorem, or the SAS Similarity Theorem to determine whether two triangles are similar. Also, students use proportions and the Triangle Proportionality Theorem or its converse to find the lengths of segments related to triangles or parallel lines. Finally, students perform dilations that are reductions or enlargements and they verify that a figure is similar to its dilation.

6 Similarity

Before

Previously, you learned the following skills, which you'll use in this chapter: using properties of parallel lines, using properties of triangles, simplifying expressions, and finding perimeter.

Prerequisite Skills

VOCABULARY CHECK

1. The alternate interior angles formed when a transversal intersects two __?__ lines are congruent. **parallel**
2. Two triangles are congruent if and only if their corresponding parts are __?__. **congruent**

SKILLS AND ALGEBRA CHECK

Simplify the expression.

3. $\frac{9 \cdot 20}{15}$ **12**
4. $\frac{15}{25}$ $\frac{3}{5}$
5. $\frac{3+4+5}{6+8+10}$ $\frac{1}{2}$
6. $\frac{2(3+8)}{77}$ $\frac{2}{7}$

Find the perimeter of the rectangle with the given dimensions.

7. $\ell = 5$ in., $w = 12$ in. **34 in.**
8. $\ell = 30$ ft, $w = 10$ ft **80 ft**
9. $A = 56\text{ m}^2$, $\ell = 8$ m **30 m**
10. Find the slope of a line parallel to the line whose equation is $y - 4 = 7(x + 2)$. **7**

Chapter Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan
- Project with Rubric

Assessment and Intervention
- Assessment Book
- Benchmark Tests
- Remediation Book
- Skills Readiness

Interactive Technology
- Power Presentations
- Activity Generator
- Animated Geometry
- ExamView™ Assessment Suite
- Online Quizzes
- eEdition
- @HomeTutor

Resources for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

Big Ideas

1. **Using ratios and proportions to solve geometry problems**
2. **Showing that triangles are similar**
3. **Using indirect measurement and similarity**

KEY VOCABULARY

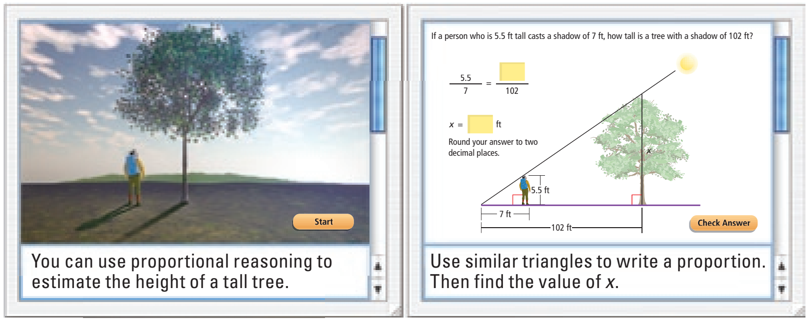
- dilation
- scale factor of a dilation
- similar polygons
- scale factor of two similar polygons
- center of dilation
- reduction
- enlargement

Why?

You can use similarity to measure lengths indirectly. For example, you can use similar triangles to find the height of a tree.

Animated Geometry

The animation illustrated below helps you answer a question from this chapter: What is the height of the tree?



Animated Geometry at my.hrw.com

Differentiated Instruction Resources

- Reading Strategies
- Differentiated Instruction Lesson Notes
- English Learners Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Using Technology in the Classroom
- Tips for New Teachers
- Math Background Notes
- Assessment Strategies
- Teacher Survival Activities
- Bulletin Board Idea

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

How student answers the exercises	What to assign from *Skills Readiness*
Any of Exs. 3–6 answered incorrectly	**Skill 10** Simplify fractions
Any of Exs. 7–9 answered incorrectly	**Skill 36** Find perimeter
Ex. 10 answered incorrectly	**Skill 76** Use slopes of parallel lines
All exercises answered correctly	Chapter Enrichment

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Solve $\frac{12}{10} = \frac{x}{60}$. **72**

2. The scale of a map is 1 cm : 10 mi. The actual distance between two towns is 4.3 miles. Find the length on the map. **0.43 cm**

3. A model train engine is 9 centimeters long. The actual engine is 18 meters long. What is the scale of the model? **1 cm : 2 m**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 0.5 block with previous lesson
0.5 block with next lesson

• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

If two figures are similar, how do you find the length of a missing side? **Tell students they will learn how to answer this question by solving a proportion using the lengths of corresponding sides.**

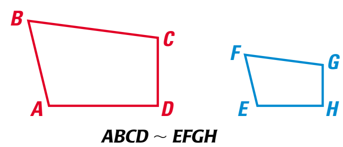
6.1 Use Similar Polygons

Before	You used proportions to solve geometry problems.
Now	You will use proportions to identify similar polygons.
Why?	So you can solve science problems, as in Ex. 34.

Key Vocabulary
- **similar polygons**
- **scale factor**

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram below, *ABCD* is similar to *EFGH*. You can write "*ABCD* is similar to *EFGH*" as $ABCD \sim EFGH$. Notice in the similarity statement that the corresponding vertices are listed in the same order.



Corresponding angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

Ratios of corresponding sides

$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

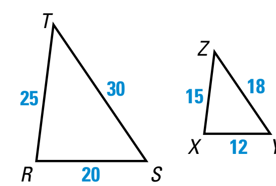
EXAMPLE 1 Use similarity statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.



READ VOCABULARY
In a *statement of proportionality*, any pair of ratios forms a true proportion.

Solution

a. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

b. $\frac{RS}{XY} = \frac{20}{12} = \frac{5}{3}$ $\quad \frac{ST}{YZ} = \frac{30}{18} = \frac{5}{3}$ $\quad \frac{TR}{ZX} = \frac{25}{15} = \frac{5}{3}$

c. Because the ratios in part (b) are equal, $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{TR}{ZX}$.

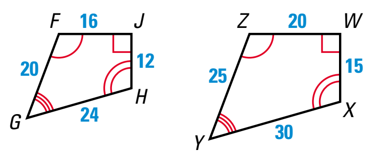
GUIDED PRACTICE for Example 1

1. Given $\triangle JKL \sim \triangle PQR$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.
$\angle J \cong \angle P$, $\angle K \cong \angle Q$, $\angle L \cong \angle R$; $\frac{JK}{PQ} = \frac{KL}{QR} = \frac{LJ}{RP}$

SCALE FACTOR If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 1, the common ratio of $\frac{5}{3}$ is the scale factor of $\triangle RST$ to $\triangle XYZ$.

EXAMPLE 2 Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.



Solution

STEP 1 **Identify** pairs of congruent angles. From the diagram, you can see that $\angle Z \cong \angle F$, $\angle Y \cong \angle G$, and $\angle X \cong \angle H$. Angles W and J are right angles, so $\angle W \cong \angle J$. So, the corresponding angles are congruent.

STEP 2 **Show** that corresponding side lengths are proportional.

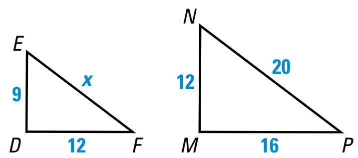
$$\frac{ZY}{FG} = \frac{25}{20} = \frac{5}{4} \qquad \frac{YX}{GH} = \frac{30}{24} = \frac{5}{4} \qquad \frac{XW}{HJ} = \frac{15}{12} = \frac{5}{4} \qquad \frac{WZ}{JF} = \frac{20}{16} = \frac{5}{4}$$

The ratios are equal, so the corresponding side lengths are proportional.

▶ So $ZYXW \sim FGHJ$. The scale factor of $ZYXW$ to $FGHJ$ is $\frac{5}{4}$.

EXAMPLE 3 Use similar polygons

xy ALGEBRA In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.



Solution

The triangles are similar, so the corresponding side lengths are proportional.

$\frac{MN}{DE} = \frac{NP}{EF}$ **Write proportion.**

$\frac{12}{9} = \frac{20}{x}$ **Substitute.**

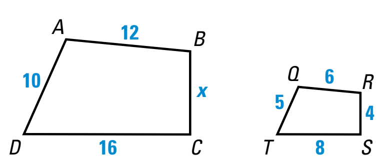
$12x = 180$ **Cross Products Property**

$x = 15$ **Solve for x.**

ANOTHER WAY There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

✓ GUIDED PRACTICE for Examples 2 and 3

In the diagram, $ABCD \sim QRST$.



2. What is the scale factor of $QRST$ to $ABCD$? $\frac{1}{2}$
3. Find the value of x. 8

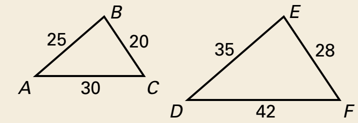
Motivating the Lesson

Tell students that a football coach uses a whiteboard to show players new plays during the game. Ask students if they think the whiteboard should look like a scale drawing of the football field.

3 TEACH

Extra Example 1

In the diagram, $\triangle ABC \sim \triangle DEF$.



a. List all pairs of congruent angles. $\angle A \cong \angle D$; $\angle B \cong \angle E$; $\angle C \cong \angle F$

b. Check that the ratios of corresponding side lengths are equal. $\frac{AB}{DE} = \frac{5}{7}$, $\frac{BC}{EF} = \frac{5}{7}$, $\frac{AC}{DF} = \frac{5}{7}$

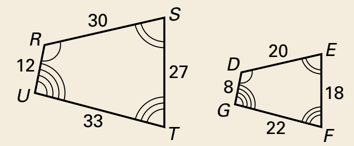
c. Write the ratios of the corresponding side lengths in a statement of proportionality. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Key Question to Ask for Example 1

- If the triangles were congruent, what would be the ratio of the corresponding sides? 1 : 1

Extra Example 2

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *RSTU* to *DEFG*.



RSTU ~ *DEFG*; the scale factor is 3 : 2.

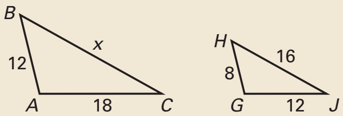
Differentiated Instruction

English Learners The word *similar* is often used in the English language to mean "alike." Similar is defined as "sharing some qualities, but not identical." Explain that in mathematics, the definition of *similar* is more specific—it describes geometric figures that differ in size, but are the same shape and have the same angle measures.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 3

In the diagram, $\triangle ABC \sim \triangle GHJ$. Find the value of x.



24

Key Question to Ask for Example 3

- What is another proportion you could write to solve the problem?

Sample answer: $\frac{12}{16} = \frac{x}{20}$

Extra Example 4

You are constructing a rectangular play area. A playground is rectangular with length 25 meters and width 15 meters. The new play area will be similar in shape, but only 10 meters in length.

a. Find the scale factor of the new play area to the playground. **2 : 5**

b. Find the perimeter of the playground and the play area. **80 m; 32 m**

ANOTHER WAY

Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:

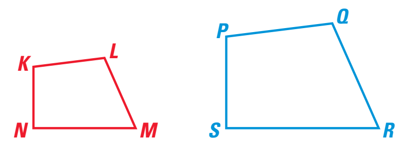
0.8(150) = 120.

PERIMETERS The ratio of lengths in similar polygons is the same as the scale factor. Theorem 6.1 shows this is true for the perimeters of the polygons.

THEOREM *For Your Notebook*

THEOREM 6.1 Perimeters of Similar Polygons

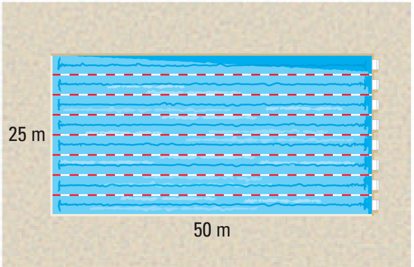
If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$.

EXAMPLE 4 Find perimeters of similar figures

SWIMMING A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.



a. Find the scale factor of the new pool to an Olympic pool.

b. Find the perimeter of an Olympic pool and the new pool.

Solution

a. Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

b. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use the Perimeters of Similar Polygons Theorem to find the perimeter x of the new pool.

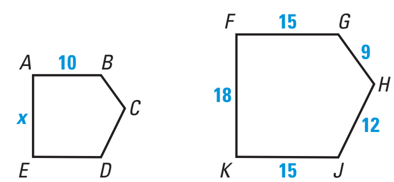
$\frac{x}{150} = \frac{4}{5}$ **Use the Perimeters of Similar Polygons Theorem to write a proportion.**

$x = 120$ **Multiply each side by 150 and simplify.**

▶ The perimeter of the new pool is 120 meters.

✓ GUIDED PRACTICE for Example 4

In the diagram, *ABCDE* ~ *FGHJK*.

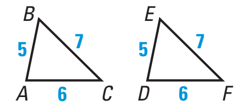


4. Find the scale factor of *FGHJK* to *ABCDE*. $\frac{3}{2}$

5. Find the value of x. **12**

6. Find the perimeter of *ABCDE*. **46**

SIMILARITY AND CONGRUENCE Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.



READ VOCABULARY
Corresponding lengths in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

CORRESPONDING LENGTHS You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

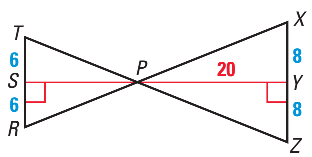
KEY CONCEPT *For Your Notebook*

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 5 Use a scale factor

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



Solution

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6+6}{8+8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ **Write proportion.**

$\frac{PS}{20} = \frac{3}{4}$ **Substitute 20 for *PY*.**

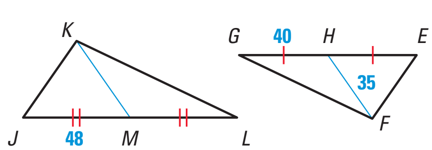
$PS = 15$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

Animated Geometry at my.hrw.com

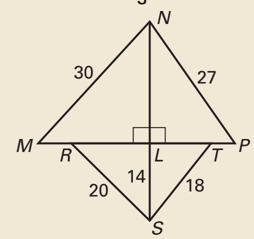
✓ GUIDED PRACTICE for Example 5

7. In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$. **42**



Extra Example 5

In the diagram, $\triangle MNP \sim \triangle RST$. Find the length of the altitude $\overline{NL}$.



21

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 5**. This activity is also part of **Power Presentations**.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: If two figures are similar, how do you find the length of a missing side?

- **Two polygons are similar if the corresponding angles are congruent and the corresponding side lengths are proportional. The ratio of corresponding lengths is the scale factor.**
- **If two polygons are similar, the ratio of any two corresponding lengths equals the scale factor.**

Write a proportion using the scale factor between the figures and the unknown side length. Then solve the proportion.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–10, 14–18, 31, 32
Day 2:
Exs. 11–13, 19–22, 33–35

Average:
Day 1:
Exs. 1–10, 14–18, 23–26, 31, 32
Day 2:
Exs. 11–13, 19–22, 27, 28, 33–36

Advanced:
Day 1:
Exs. 1, 2, 4–10, 14–18, 23–26, 29–32*
Day 2:
Exs. 11, 12, 19–22, 27, 28, 33–39*

Block:
Exs. 1–10, 14–18, 23–26, 31, 32 (with previous lesson)
Exs. 11–13, 19–22, 27, 28, 33–36 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 8, 11, 21, 31
Average: 5, 9, 12, 22, 31
Advanced: 6, 10, 11, 22, 32

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

6.1 EXERCISES

HOMEWORK KEY

○ = See WORKED-OUT SOLUTIONS Exs. 3, 7, and 31

★ = STANDARDIZED TEST PRACTICE Exs. 2, 6, 18, 27, 28, 35, 36, and 37

◆ = MULTIPLE REPRESENTATIONS Ex. 33

SKILL PRACTICE

A

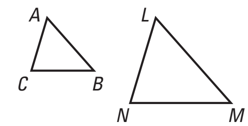
1. VOCABULARY Copy and complete: Two polygons are similar if corresponding angles are _?_ and corresponding side lengths are _?_.
congruent, proportional

2. ★ WRITING If two polygons are congruent, must they be similar? If two polygons are similar, must they be congruent? *Explain.* **See margin.**

EXAMPLE 1 for Exs. 3–6

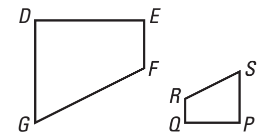
USING SIMILARITY **List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.**

3. $\triangle ABC \sim \triangle LMN$



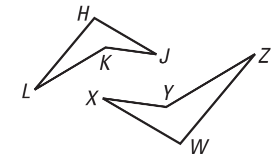
3. $\angle A \cong \angle L$, $\angle B \cong \angle M$, $\angle C \cong \angle N$; $\frac{AB}{LM} = \frac{BC}{MN} = \frac{CA}{NL}$

4. $DEFG \sim PQRS$



4. $\angle D \cong \angle P$, $\angle E \cong \angle Q$, $\angle F \cong \angle R$, $\angle G \cong \angle S$; $\frac{DE}{PQ} = \frac{EF}{QR} = \frac{FG}{RS} = \frac{GD}{SP}$

5. $HJKL \sim WXYZ$



5. $\angle H \cong \angle W$, $\angle J \cong \angle X$, $\angle K \cong \angle Y$, $\angle L \cong \angle Z$; $\frac{HJ}{WX} = \frac{JK}{XY} = \frac{KL}{YZ} = \frac{LH}{ZW}$

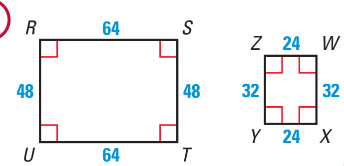
6. ★ MULTIPLE CHOICE Triangles *ABC* and *DEF* are similar. Which statement is *not* correct? **D**

Ⓐ $\frac{BC}{EF} = \frac{AC}{DF}$ Ⓑ $\frac{AB}{DE} = \frac{CA}{FD}$ Ⓒ $\frac{CA}{FD} = \frac{BC}{EF}$ Ⓓ $\frac{AB}{EF} = \frac{BC}{DE}$

EXAMPLES 2 and 3 for Exs. 7–10

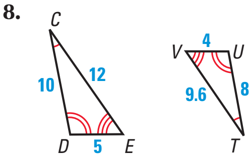
DETERMINING SIMILARITY **Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.**

7.



similar; $RSTU \sim WXYZ$, $\frac{2}{1}$

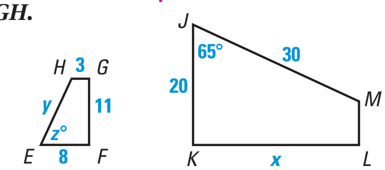
8.



similar; $\triangle CDE \sim \triangle TUV$, $\frac{5}{4}$

EXAMPLE 4 for Exs. 11–13

USING SIMILAR POLYGONS **In the diagram, $JKLM \sim EFGH$.**



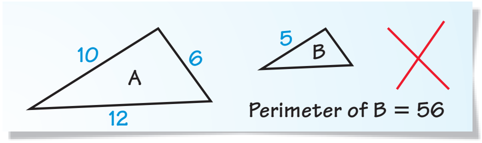
9. Find the scale factor of *JKLM* to *EFGH*. $\frac{5}{2}$

10. Find the values of *x*, *y*, and *z*. **27.5, 12, 65**

11. Find the perimeter of each polygon. **85, 34**

12. PERIMETER Two similar FOR SALE signs have a scale factor of 5 : 3. The large sign's perimeter is 60 inches. Find the small sign's perimeter. **36 in.**

13. ERROR ANALYSIS The triangles are similar. *Describe* and correct the error in finding the perimeter of Triangle B. **The larger triangle's perimeter was doubled but should have been halved; perimeter of $B = 14$.**



2. Yes; no; if two polygons are congruent, corresponding angles are congruent and the scale factor is 1. If they are similar, corresponding angles are congruent but one polygon can be larger than the other since the scale factor does not have to be 1.

B **REASONING** **Are the polygons *always*, *sometimes*, or *never* similar?**

14. Two isosceles triangles **sometimes**

15. Two equilateral triangles **always**

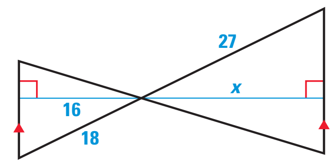
16. A right triangle and an isosceles triangle **sometimes**

17. A scalene triangle and an isosceles triangle **never**

18. ★ **SHORT RESPONSE** The scale factor of Figure A to Figure B is $1:x$. What is the scale factor of Figure B to Figure A? *Explain* your reasoning.
$x:1$; since the order of the figures switched, simply switch the ratio.

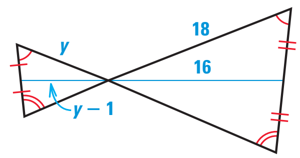
SIMILAR TRIANGLES **The black triangles are similar. Identify the type of special segment shown in blue, and find the value of the variable.**

19.



altitude, 24

20.



median, 9

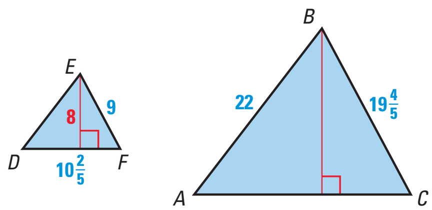
EXAMPLE 5
for Exs. 21–22

USING SCALE FACTOR **Triangles *NPQ* and *RST* are similar. The side lengths of $\triangle NPQ$ are 6 inches, 8 inches, and 10 inches, and the length of an altitude is 4.8 inches. The shortest side of $\triangle RST$ is 8 inches long.**

21. Find the lengths of the other two sides of $\triangle RST$. **$10\frac{2}{3}$ in., $13\frac{1}{3}$ in.**

22. Find the length of the corresponding altitude in $\triangle RST$. **6.4 in.**

USING SIMILAR TRIANGLES **In the diagram, $\triangle ABC \sim \triangle DEF$.**



23. Find the scale factor of $\triangle ABC$ to $\triangle DEF$. **$\frac{11}{5}$**

24. Find the unknown side lengths in both triangles. **$AC = 22\frac{22}{25}$, $ED = 10$**

25. Find the length of the altitude shown in $\triangle ABC$. **$17\frac{3}{5}$**

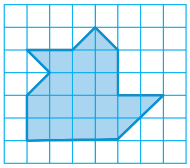
26. Find and compare the areas of both triangles.
About 201, 41.6; the ratio of their areas is approximately equal to the scale factor squared.

27. ★ **SHORT RESPONSE** Suppose you are told that $\triangle PQR \sim \triangle XYZ$ and that the extended ratio of the angle measures in $\triangle PQR$ is $x : x + 30 : 3x$. Do you need to know anything about $\triangle XYZ$ to be able to write its extended ratio of angle measures? *Explain* your reasoning.
No; in similar triangles corresponding angles are congruent.

28. ★ **MULTIPLE CHOICE** The lengths of the legs of right triangle *ABC* are 3 feet and 4 feet. The shortest side of $\triangle UVW$ is 4.5 feet and $\triangle UVW \sim \triangle ABC$. How long is the hypotenuse of $\triangle UVW$? **D**

(A) 1.5 ft (B) 5 ft (C) 6 ft (D) 7.5 ft

C 29. **CHALLENGE** Copy the figure at the right and divide it into two similar figures. **See margin.**



30. **REASONING** Is similarity reflexive? symmetric? transitive? Give examples to support your answers.
Yes, yes, yes. *Sample answer:* $\triangle ABC \sim \triangle ABC$.
If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.
If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle HJK$, then $\triangle ABC \sim \triangle HJK$.

Avoiding Common Errors

Exercise 6 If students have difficulty with this exercise, remind them to match up the letters corresponding to the triangles in the order they are written. For example, *AB* uses the first two letters of $\triangle ABC$, so its corresponding side uses the first two letters of $\triangle DEF$.

Exercises 7–8 If students have difficulty with these exercises, suggest that they redraw the diagrams so that corresponding letters appear in corresponding positions.

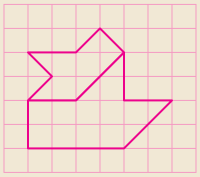
Teaching Strategy

Exercises 14–17 Encourage students to draw examples to verify their answers to these exercises. Remind them that one counterexample eliminates the "always" choice. Suggest that they start by looking for counterexamples.

Mathematical Reasoning

Exercise 26 For a pair of similar figures, including a pair formed from an enlargement or reduction (a dilation), if the scale factor is $\frac{a}{b}$, then the corresponding sides, related segments, and perimeters all have the same scale factor, $\frac{a}{b}$, while the area has scale factor $\frac{a^2}{b^2}$.

29.



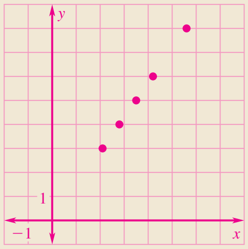
Study Strategy

Exercise 35 Encourage students to analyze this problem by substituting numbers for the variables and then using algebra to prove their conclusion.

Internet Reference

Exercise 34 Additional information about solar eclipses can be found at csep10.phys.utk.edu/astr161/lect/time/eclipses.html

33b.

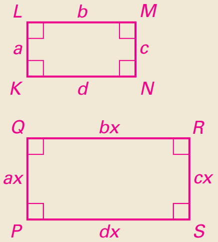


34a. See below.

37b. $\angle BOA \cong \angle DOC$ **by the Vertical Angles Theorem;** $\angle OBA \cong \angle ODC$ **by the Alternate Interior Angles Theorem;** $\angle BAO \cong \angle DCO$ **by the Alternate Interior Angles Theorem.**

37c. $A(-3, 0)$, $B(0, 4)$, $C(6, 0)$, $D(0, -8)$; $AO = 3$, $OB = 4$, $BA = 5$, $CO = 6$, $OD = 8$, $DC = 10$

38. ***Sample answer:*** $KLMN \sim PQRS$ **is given. Since the rectangles are similar, let** $\frac{x}{1}$ **be the scale factor and let** a, b, c, d **be the lengths of the sides of** $KLMN$ **and** ax, bx, cx, dx **be the lengths of the corresponding sides of** $PQRS$**. Taking the ratio of perimeters you get** $\frac{ax + bx + cx + dx}{a + b + c + d} = \frac{x(a + b + c + d)}{a + b + c + d} = x.$



Problem Solving

EXAMPLE 2 A
Exs. 31–32

31. TENNIS In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? *Explain.* If so, find the scale factor of the tennis court to the table.

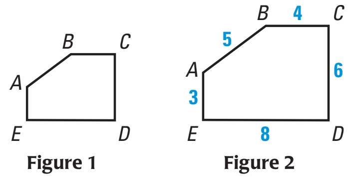
No; the lengths are not proportional.

32. DIGITAL PROJECTOR You are preparing a computer presentation to be digitally projected onto the wall of your classroom. Your computer screen is 13.25 inches wide and 10.6 inches high. The projected image on the wall is 53 inches wide and 42.4 inches high. Are the two shapes similar? If so, find the scale factor of the computer screen to the projected image. **yes;** $\frac{1}{4}$

33. MULTIPLE REPRESENTATIONS Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7 : 10.

a. Making a Table Copy and complete the table.

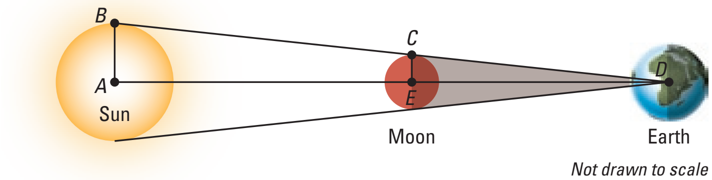
	AB	BC	CD	DE	EA
Figure 1	3.5	? 2.8	? 4.2	? 5.6	? 2.1
Figure 2	5.0	4.0	6.0	8.0	3.0



b. Drawing a Graph Graph the data in the table. Let x represent the length of a side in Figure 1 and let y represent the length of the corresponding side in Figure 2. Is the relationship linear? **See margin for art; yes.**

c. Writing an Equation Write an equation that relates x and y. What is its slope? How is the slope related to the scale factor? $y = \frac{10}{7}x$; $\frac{10}{7}$; **they are the same.**

34. MULTI-STEP PROBLEM During a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The distance ED between Earth and the moon is 240,000 miles, the distance DA between Earth and the sun is 93,000,000 miles, and the radius AB of the sun is 432,500 miles.



a. Copy the diagram and label the known distances. **See margin.**

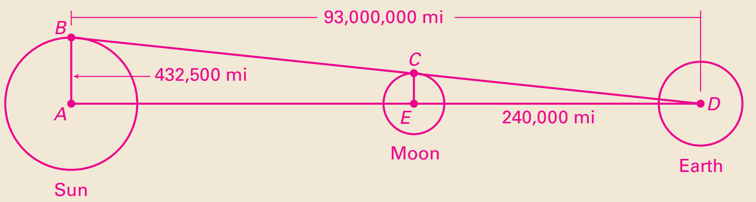
b. In the diagram, $\triangle BDA \sim \triangle CDE$. Use this fact to explain a total eclipse of the sun.

c. Estimate the radius CE of the moon. **about 1116 mi**

34b. ***Sample answer:*** **Since the triangles are similar, the light from every point of the sun that is in** $\triangle BDA$ **is blocked by the moon before reaching Earth.**

○ = See WORKED-OUT SOLUTIONS in Student Resources
★ = STANDARDIZED TEST PRACTICE
= MULTIPLE REPRESENTATIONS

34a.

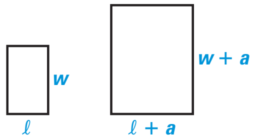


35. Yes; if $\ell = w$ then the larger and smaller image would be similar. *Sample answer:* Let $\ell = 8$, $w = 8$, and $a = 4$; $\frac{w}{w+a} = \frac{8}{12} = \frac{2}{3}$, $\frac{\ell}{\ell+a} = \frac{8}{12} = \frac{2}{3}$.

36. The ratio of the areas of similar figures is the square of the scale factor. *Sample answer:* Consider two rectangles, one 2x4 and the other 6x12, with scale factor $\frac{1}{3}$ and ratio of areas $\frac{8}{72} = \frac{1}{9}$, $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$.

B

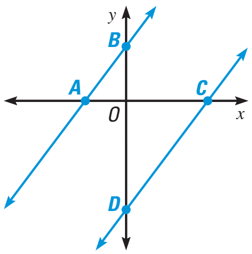
35. ★ **SHORT RESPONSE** A rectangular image is enlarged on each side by the same amount. The angles remain unchanged. Can the larger image be similar to the original? *Explain* your reasoning, and give an example to support your answer.



36. ★ **SHORT RESPONSE** How are the areas of similar rectangles related to the scale factor? Use examples to *justify* your reasoning.

C

37. ★ **EXTENDED RESPONSE** The equations of two lines in the coordinate plane are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$.



a. *Explain* why the two lines are parallel. They have the same slope.

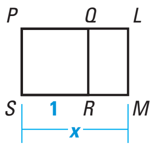
b. Show that $\angle BOA \cong \angle DOC$, $\angle OBA \cong \angle ODC$, and $\angle BAO \cong \angle DCO$. b, c. See margin.

c. Find the coordinates of points A, B, C, and D. Find the lengths of the sides of $\triangle AOB$ and $\triangle COD$.

d. Show that $\triangle AOB \sim \triangle COD$. Since corresponding angles are congruent and the ratios of corresponding sides are all the same the triangles are similar.

38. **PROVING THEOREM 6.1** Prove the Perimeters of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof. See margin.

39. **CHALLENGE** In the diagram, $PQRS$ is a square, and $PLMS \sim LMRQ$. Find the exact value of x. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles. See margin.

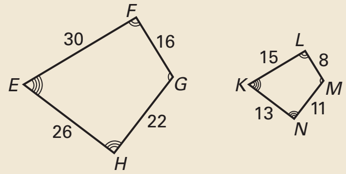


5 ASSESS AND RETEACH

Daily Homework Quiz

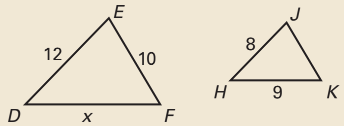
Also available online

1. Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *EFGH* to *KLMN*.



yes; $EFGH \sim KLMN$; the scale factor is 2 : 1.

2. In the diagram, $\triangle DEF \sim \triangle HJK$. Find the value of x.



13.5

3. Two similar triangles have the scale factor 5 : 4. Find the ratio of their corresponding altitudes and medians. 5 : 4; 5 : 4

4. Two similar triangles have the scale factor 3 : 7. Find the ratio of their corresponding perimeters and areas. 3 : 7; 9 : 49

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

1 PLAN AND PREPARE

Explore the Concept

- Students will discover the properties of dilations by using graph paper to dilate a figure.

Materials

Each student will need:

- graph paper
- ruler
- protractor

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Key Questions

- In the Explore, to what segments are $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ transformed? **$\overline{AD}$, $\overline{AE}$, and $\overline{DE}$**
- Which segment and its image appear to be parallel? **$\overline{BC}$ and $\overline{DE}$**

Key Discovery

A dilation takes a line not passing through the center of dilation to a parallel line, and lines passing through the center are unchanged. The lengths of segments are changed by a ratio called the scale factor. All dilations preserve angle measures.

3 ASSESS AND RETEACH

1. A dilation with scale factor 1.5 and center (0, 0) is applied to a triangle with sides of lengths 1, 1, and $\sqrt{2}$. Describe the longest side of the image. **Its length is $1.5\sqrt{2}$ and it is parallel to the longest side of the preimage.**

Explore Properties of Dilations

MATERIALS • graph paper • ruler • protractor

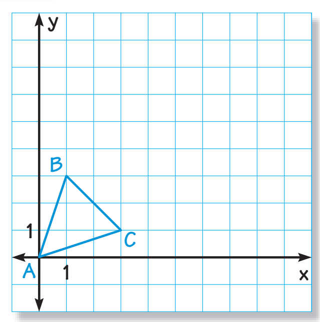
QUESTION How do dilations affect lines, segments, and angles?

Dilations are non-rigid transformations that map points to points in the coordinate plane. A function rule in coordinate notation for a dilation with center at the origin is $(x, y) \to (kx, ky)$.

The ratio of the length of an image segment to its corresponding preimage segment is the *scale factor* k of the dilation.

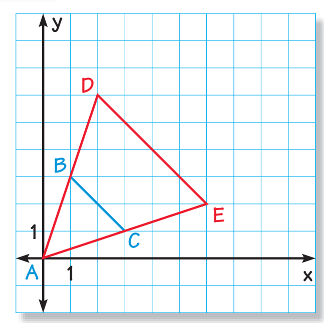
EXPLORE Draw a dilation of a triangle in the plane

STEP 1



Graph a triangle Graph a triangle in a coordinate plane one of whose vertices is (0, 0).

STEP 2



Dilate the triangle Graph a dilation of the triangle with scale factor 2 and centered at the origin.

DRAW CONCLUSIONS Use your observations to complete these exercises **1–5. See margin.**

1. Measure the sides and angles of $\triangle ABC$ and $\triangle ADE$. Tell whether the dilation preserves lengths or angle measures.
2. Compare the ratios $\frac{DE}{BC}$, $\frac{DA}{BA}$, and $\frac{EA}{CA}$. What do you notice?
3. What is the effect of the dilation on the center of dilation? *Explain.*
4. How is $\overleftrightarrow{AB}$ related to $\overleftrightarrow{AD}$? How does a dilation affect a line that passes through the center of dilation? *Explain.*
5. How is $\overleftrightarrow{BC}$ related to $\overleftrightarrow{DE}$? How does a dilation affect a line that does not pass through the center of dilation? *Explain.*

1–5. Sample answers are given.

1. $AB = \sqrt{10}$, $AC = \sqrt{10}$, $BC = 2\sqrt{2}$, $AD = 2\sqrt{10}$, $AE = 2\sqrt{10}$, $DE = 4\sqrt{2}$; $m\angle A \approx 53°$, $m\angle B = m\angle D \approx 63.5°$, $m\angle C = m\angle E \approx 63.5°$; **the dilation does not preserve lengths but it does preserve angle measures.**

2. The ratios are all equal to the scale factor, 2.

3. The center of dilation (0, 0) is mapped to $(k \cdot 0, k \cdot 0) = (2 \cdot 0, 2 \cdot 0) = (0, 0)$. The center of dilation is mapped to itself.

4. The lines are the same; the image of a line that passes through the center of dilation is the same as the preimage line; the center of dilation is mapped to itself, and any line that contains the origin is mapped to a line that contains the origin.

5. The lines are parallel; the image of a line that does not pass through the center of dilation is a line parallel to the preimage line; because $\angle C \cong \angle E$, the lines are parallel by the Corresponding Angles Converse.

6.2 Relate Transformations and Similarity

Before You identified rigid motions in the plane.

Now You will identify similarity transformations called dilations.

Why? So you can find the dimensions of a scale drawing, as in Ex. 30.

Key Vocabulary
- dilation
- scale factor

A **dilation** is a transformation that preserves angle measures and results in an image with lengths proportional to the preimage lengths.

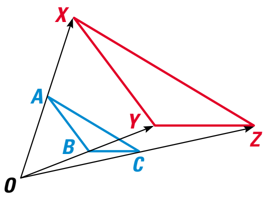
The ratio of the lengths of the corresponding sides of the image and the preimage is called the **scale factor** of the dilation. Dilations can also be called *similarity transformations.*

KEY CONCEPT *For Your Notebook*

Dilations and Similarity

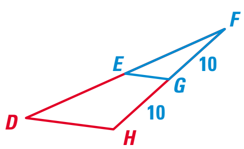
If a dilation can be used to move one figure onto another, the two figures are similar.

$\triangle ABC \sim \triangle XYZ$



EXAMPLE 1 **Describe a dilation**

$\triangle FEG$ is similar to $\triangle FDH$. Describe the dilation that moves $\triangle FEG$ onto $\triangle FDH$.



Solution

The figure shows a dilation with center *F*.

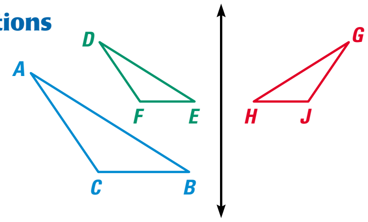
The scale factor is 2 because the ratio of *FH* to *FG* is 20 : 10, or 2 : 1.

KEY CONCEPT *For Your Notebook*

Combining Dilations and Rigid Motions

If a dilation followed by any combination of rigid motions can be used to move one figure onto the other, the two figures are similar.

$\triangle ABC \sim \triangle DEF$ and $\triangle DEF \cong \triangle GHJ$, so $\triangle ABC \sim \triangle GHJ$.



1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Given $\triangle RST \sim \triangle XYZ$ with $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{3}{2}$. Find $\frac{RT}{XZ}$. $\frac{3}{2}$

2. Given $\triangle EFG \sim \triangle MNP$ with $\frac{EF}{MN} = \frac{2}{1}$. If $FG = 4.5$, find NP. **2.25**

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How do you identify a similarity transformation in the plane? **Tell students they will learn how to answer this question by examining dilations, transformations in the plane that preserve angle measure while keeping the lengths of corresponding sides proportional.**

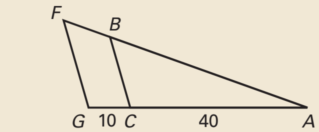
Motivating the Lesson

Ask students to draw examples of figures that are similar based on their recollection from previous work. Have them point out any examples of similar figures they see in the classroom.

3 TEACH

Extra Example 1

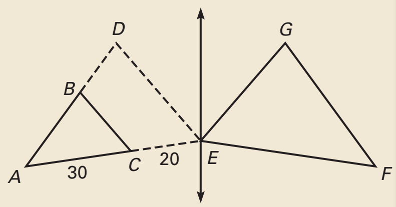
$\triangle ABC$ is similar to $\triangle AFG$. Describe the dilation that moves $\triangle ABC$ onto $\triangle AFG$.



dilation with center *A* and scale factor $\frac{5}{4}$

Extra Example 2

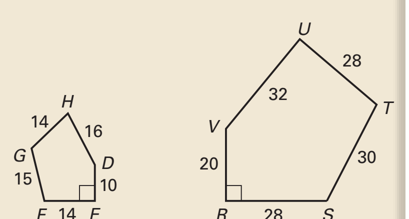
$\triangle ABC$ is similar to $\triangle FGE$. Describe a combination of transformations that moves $\triangle ABC$ onto $\triangle FGE$.



A dilation with center *A* and scale factor $\frac{5}{3}$ moves $\triangle ABC$ onto $\triangle ADE$. Then a reflection moves $\triangle ADE$ onto $\triangle FGE$.

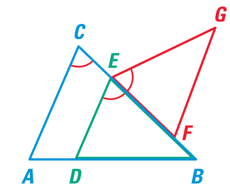
Extra Example 3

Use transformations to explain why pentagons *EFGHD* and *RSTUV* are *not* similar.



EXAMPLE 2 Describe a combination of transformations

$\triangle ABC$ is similar to $\triangle FGE$. Describe a combination of transformations that moves $\triangle ABC$ onto $\triangle FGE$.



Solution

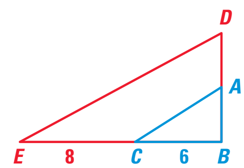
A dilation with center B and scale factor $\frac{2}{3}$ moves $\triangle ABC$ onto $\triangle DBE$.

Then a rotation of $\triangle DBE$ with center E moves $\triangle DBE$ onto $\triangle FGE$. The angle of rotation is equal to the measure of $\angle C$.

GUIDED PRACTICE for Examples 1 and 2

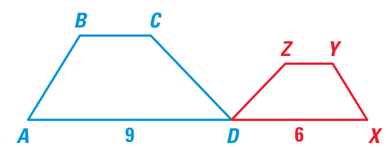
The two figures are similar. Describe the transformation(s) that move the blue figure onto the red figure.

1.



dilation with center *B* and scale factor $\frac{7}{3}$

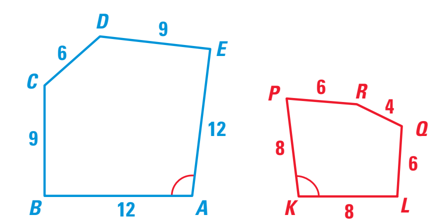
2.



dilation with scale factor $\frac{2}{3}$ and reflection

EXAMPLE 3 Use transformations to show figures are not similar

Use transformations to explain why *ABCDE* and *KLQRP* are *not* similar.

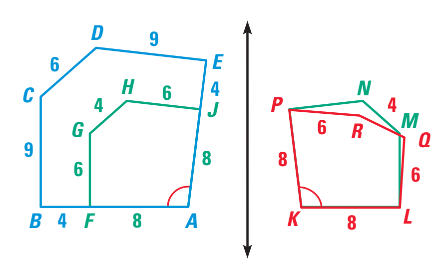


Solution

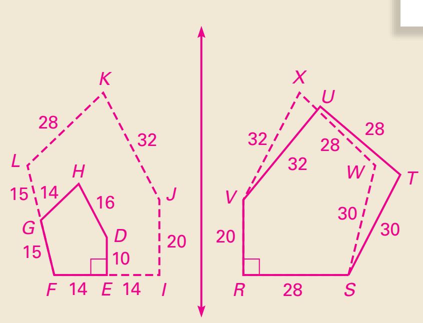
Corresponding sides in the pentagons are proportional with a scale factor of $\frac{2}{3}$.

However, this does not necessarily mean the pentagons are similar.

A dilation with center A and scale factor $\frac{2}{3}$ moves *ABCDE* onto *AFGHJ*. Then a reflection moves *AFGHJ* onto *KLMNP*.



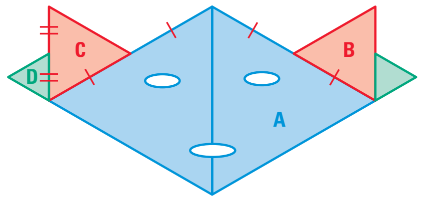
KLMNP does not exactly coincide with *KLQRP*, because not all of the corresponding angles are congruent. (Only $\angle A$ and $\angle K$ are congruent.) Since angle measure is not preserved, the two pentagons are not similar.



Corresponding sides in the pentagons are proportional with a scale factor of 2. However, a dilation with center *F* moves *EFGHD* onto *IFLKJ*. Then a reflection moves *IFLKJ* onto *RSWXV*. But *RSWXV* does not exactly coincide with *RSTUV*, since not all of the corresponding angles are congruent. Since angle measure is not preserved, the two pentagons are not similar.

EXAMPLE 4 Use similar figures

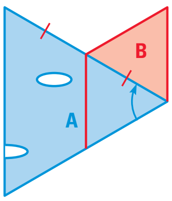
GRAPHIC DESIGN **A design for a party mask is made using all equilateral triangles and a scale factor of $\frac{1}{2}$.**



a. Describe transformations that move triangle A onto triangle B.

b. Describe why triangles C and D are similar by using the given infomation.

Solution

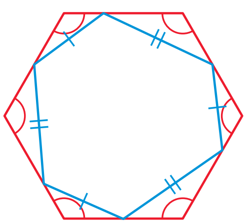


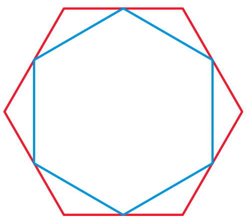
a. The figure shows a dilation with scale factor $\frac{1}{2}$ followed by a clockwise rotation of 60°.

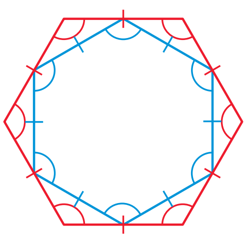
b. Triangles C and D are similar because all pairs of corresponding sides are proportional with a ratio of $\frac{1}{2}$ and all pairs of corresponding angles of equilateral triangles have the same measure.

GUIDED PRACTICE for Examples 3 and 4

Refer to the floor tile designs shown below. In each design, the red shape is a regular hexagon. 3–6. See margin.


Tile design 1


Tile design 2


Tile design 3

3. Tile design 1 is made using two hexagons. *Explain* why the red and blue hexagons are not similar.

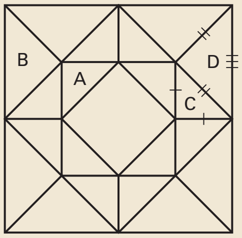
4. Tile design 2 is made using two similar geometric shapes. *Describe* the transformations that move the blue hexagon to the red hexagon.

5. Tile design 3 shows congruent angles and sides. *Explain* why the red and blue hexagons are similar, using the given information.

6. If the lengths of all the sides of one polygon are proportional to the lengths of all the corresponding sides of another polygon, must the polygons be similar? *Explain*.

Extra Example 4

The design for a stained glass window uses two sizes of similar isosceles right triangles with a scale factor of $\sqrt{2}$.



a. Describe the transformations that move triangle A onto triangle B. **A dilation with scale factor $\sqrt{2}$ centered at the vertex of the right angle in triangle A followed by a clockwise rotation of 135° about that same point will move triangle A onto triangle B.**

b. Describe why triangles C and D are similar using the given information. **Triangles C and D are similar because all pairs of corresponding sides are proportional with a ratio of $\sqrt{2}$ to 1 and corresponding angles of isosceles right triangles have the same measure (45° or 90°).**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you identify a similarity transformation in the plane?

- **A dilation is a transformation that preserves angle measures and results in an image with lengths proportional to the preimage lengths.**
- **Dilations are similarity transformations.**

If a dilation or a dilation followed by any combination of rigid motions can move one figure onto another, then the two figures are similar.

3–6. Sample answers are given.

3. **The red hexagon has all sides congruent, but the blue hexagon has 3 shorter sides and 3 longer sides, so ratios of corresponding side lengths are not constant.**

4. **dilation followed by a rotation of 30° about the center of the figures**

5. **All angles are congruent, so angle measure is preserved, and all side lengths are congruent in each hexagon, so the ratio of any two corresponding side lengths is constant.**

6. **No; even though corresponding sides might be proportional, if corresponding angles are not congruent, the polygons are not similar.**

4 PRACTICE AND APPLY

Assignment Guide

Basic:
Day 1:
Exs. 1, 3, 7–11, 13, 15, 18, 25–27
Average:
Day 1:
Exs. 1–10, 11–25 odd, 26, 28–30, 32
Advanced:
Day 1:
Exs. 4–9, 12–16 even, 20–31, 32
Block:
Exs. 1–10, 11–25 odd, 26, 28–30, 32

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:
Basic: 3, 8, 13, 18, 25
Average: 5, 9, 15, 19, 25
Advanced: 5, 8, 16, 20, 26

Extra Practice

• Practice B in Chapter Resources

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

2. *Sample answer:* A similarity transformation maps one figure onto a similar figure. The corresponding sides have lengths that are proportional and the corresponding angles have the same measures.

6. The function notation is for a dilation with scale factor 3. Corresponding sides will be proportional with a ratio of 3 to 1. Choosing a sample figure and drawing its image will show that the corresponding angles of the figures have the same measure.

6.2 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 15, 19, and 27
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 10, 17, 26, and 30

SKILL PRACTICE

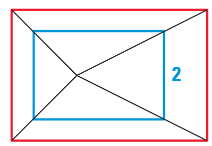
A

1. **VOCABULARY** Draw an example of a dilation.
Check students' drawings.

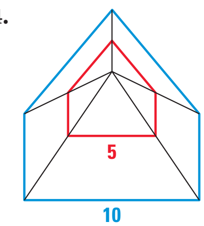
2. ★ **WRITING** Describe the results of a similarity transformation. See margin.

EXAMPLE 1 for Exs. 1–6

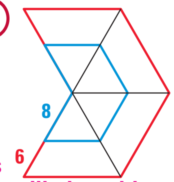
DESCRIBING DILATIONS **Describe the dilation that moves the blue figure onto the red figure.**

3.


dilation with scale factor $\frac{3}{2}$ and center at intersection of black lines

4.


dilation with scale factor $\frac{1}{2}$ and center at intersection of black lines

5.


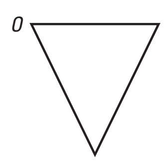
dilation with scale factor $\frac{14}{8} = \frac{7}{4}$ and center at intersection of black lines

6. ★ **WRITING** Jenny described a transformation in the coordinate plane using the notation $(x, y) \rightarrow (3x, 3y)$. Explain why her transformation is a dilation with center at the origin. See margin.

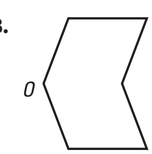
EXAMPLE 2 for Exs. 7–9

DRAWING SIMILARITY TRANSFORMATIONS **Copy the figure. Draw an example of the given similarity transformation of the figure with center *O*.**

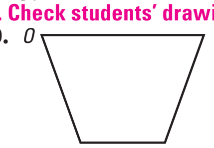
7–9. Check students' drawings.

7.


dilation

8.


dilation then reflection

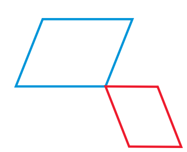
9.


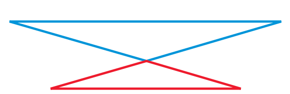
dilation then rotation

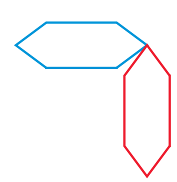
EXAMPLE 3 for Ex. 10–12

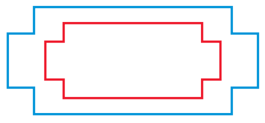
B

10. ★ **MULTIPLE CHOICE** Which of the following transformations does *not* involve dilation? C

(A)


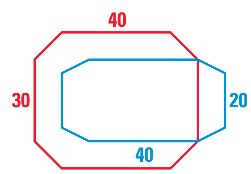
(B)


(C)


(D)


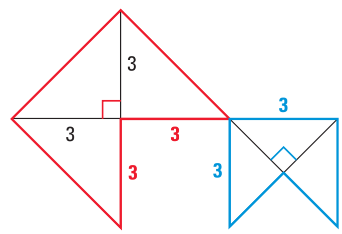
IDENTIFYING DILATIONS **The red figure is the image of the blue figure under a transformation. Tell whether the transformation involves a dilation. If so, give the scale factor of the dilation.**

11.



no

12.



yes; dilation with scale factor $\sqrt{2}$

COMBINING TRANSFORMATIONS **Coordinates of the vertices of a preimage and image figure are given. Describe the transformations that move the first figure onto the second.** 13–16. See margin.

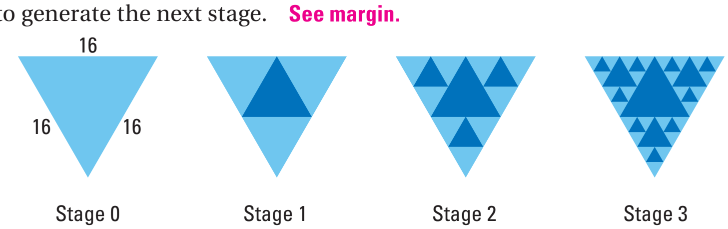
13. $O(0, 0)$, $B(0, 3)$, $C(2, 0)$; $O(0, 0)$, $D(0, -6)$, $E(4, 0)$

14. $O(0, 0)$, $P(0, 8)$, $Q(6, 0)$; $O(0, 0)$, $R(4, 0)$, $S(0, -3)$

15. $J(-4, 0)$, $K(0, 4)$, $L(6, 0)$; $L(6, 0)$, $M(0, 6)$, $N(-9, 0)$

16. $A(-6, 0)$, $B(0, 6)$, $C(9, 0)$, $D(0, -15)$; $W(0, -2)$, $X(-2, 0)$, $Y(0, 3)$, $Z(5, 0)$

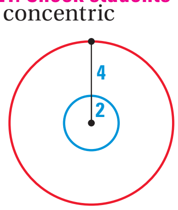
17. ★ **SHORT RESPONSE** A fractal called the *Sierpinski triangle* can be imagined by thinking of an infinite sequence of stages, the first of which are shown below. Calculate the side lengths of the light blue triangles in Stages 1, 2, and 3. Then describe the 3 dilations you can apply to any stage to generate the next stage. See margin.



SIMILARITY OF CIRCLES **Prove the circles are similar by finding a center and scale factor of a dilation that moves the blue circle onto the red circle.**
18–21. Check students' drawings.

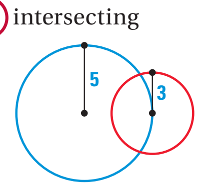
18. concentric

scale factor 3



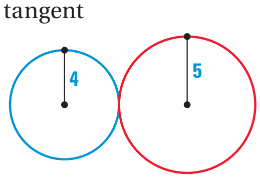
19. intersecting

scale factor $\frac{3}{5}$



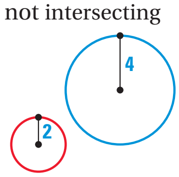
20. tangent

scale factor $\frac{5}{4}$



21. not intersecting

scale factor $\frac{1}{2}$



Teaching Strategy

Exercise 10 You might wish to ask students to explain the transformation that moves one figure onto the other in each of the answer choices.

Avoiding Common Errors

Exercises 13–16 While some students may try to visualize the figures mentally, urge all students to graph the points on a coordinate grid so they can visualize the transformations that move the first figure onto the second figure.

Study Strategy

Exercise 17 Suggest that students make a table of the side lengths of the triangle(*s*) at each stage. Doing so will help them determine the ratio of the corresponding side lengths.

13. dilation with center *O* and scale factor 2, then reflection in the *x*-axis
14. dilation with center *O* and scale factor $\frac{1}{2}$, then rotation 90° clockwise around *O*
15. dilation with center *O* and scale factor $\frac{3}{2}$, then reflection in the *y*-axis
16. dilation with center *O* and scale factor $\frac{1}{3}$, then rotation 90° counterclockwise around *O*
17. *Sample answer:* The lengths are 8, 4, and 2; dilate the previous stage 3 times with scale factor $\frac{1}{2}$ using each corner of the triangle as a center to generate the next stage.

Differentiated Instruction

Below Level For Exercises 13–16, have students use a large sheet of grid paper to graph each pair of figures. Have them measure the sides and angles of the preimage and of the image. Ask students to find the ratio of the corresponding side lengths and to compare the corresponding angle measures.

See also the *Differentiated Instruction Resources* for more strategies.

Study Strategy

Exercise 23 Suggest that students draw a diagram to model this situation and to help them formulate their explanation.

Teaching Strategy

Exercise 26 You may want students to share and discuss their designs with a partner. Have them describe the transformations they used, trade their designs with their partner, and then have the partner also describe the transformations. They can then finish the discussion by comparing their descriptions.

22a. The perimeter of the image is 4 times the perimeter of the preimage.

22b. The area of the image is 16 times the area of the preimage.

23. *Sample answer:* Because the purses are similar, the designs of the purses should have the same shape but not the same size. Use a copy machine to enlarge the pattern from the smaller purse. For a purse twice as big, use a setting of 200% on the copy machine. Then transfer the pattern to the larger purse.

24. *Sample answer:* An overhead projector enlarges a figure onto a screen as a function of the distance from the projector to the screen. In place of the screen, affix a poster board. The pattern can be traced onto the board.

29. *Sample answer:* Draw corresponding radii in the circles parallel to each other. Draw a line through the endpoints of the radii that lie on the circles. Draw a line through the centers of the circles. The center of dilation is the intersection of the lines. The scale factor of the dilation is the ratio of the radii of the circles.

C **22. CHALLENGE** The dilation of a rectangle has a scale factor of 4 to 1. **See margin.**

a. Describe the effect of the dilation on the perimeter of the rectangle.

b. Describe the effect of the dilation on the area of the rectangle.

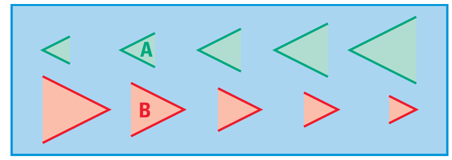
PROBLEM SOLVING

EXAMPLE 4 A for Exs. 23–28

23. TEXTILES A cloth purse maker uses a pattern for a small bag. The maker wants to start making a similar bag that is exactly twice as big as the small one and has the same design. Explain how you might efficiently create the design for the large bag. **See margin.**

24. GRAPHIC ARTS Describe how using an overhead projector involves dilations when creating a larger image that can be traced onto a poster. **See margin.**

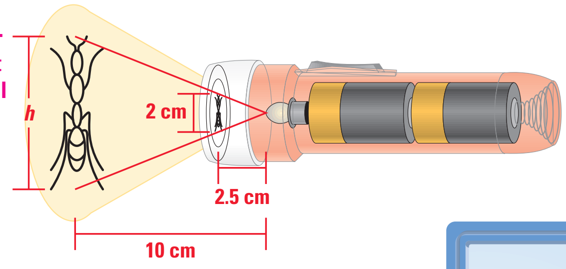
25. CRAFTS A rug design uses similar triangles that become larger on one side while getting smaller on the other side. What transformations are used to move figure A onto figure B? **dilation followed by a rotation**



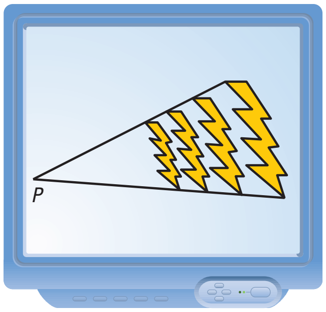
B **26. ★ OPEN-ENDED** Create a design using a combination of dilations and other transformations. **Check students' designs.**

27. TOY IDEA A flashlight has a removable cap with a picture on it. When the flashlight is on, the bug can be projected onto a wall. Using the information in the diagram below, find the scale factor of the dilation. Then calculate the height of the bug projected onto the wall.

The scale factor of the dilation is $\frac{10}{25} = 4$. Therefore, the height of the bug on the wall will be 4 times the height of the bug on the flashlight cap; $4 \cdot 2$ cm $= 8$ cm.



28. ANIMATION You are designing an animated lightning bolt on a computer screen. You want the distance of each bolt from P to be $\frac{1}{5}$ greater than the distance of the previous bolt. Describe the transformation that moves each bolt to the next larger bolt. **dilation with scale factor $\frac{6}{5}$**



29. CHALLENGE Describe a method for finding the center and scale factor of a dilation that moves a given circle onto another given circle with a different radius. **See margin.**

30a. *Sample answer:* The floor plan is a model. Each part of the actual house will copy the floor plan and be a dilation of the shape. The final size of every inch on the scale drawing will be 2 feet, or 24 inches, so the scale factor is 24 to 1.

30b. *Sample answer:* Multiply each dimension on the floor plan by 24 to find the actual size in inches; divide by 12 to find the actual size in feet. So, the living room that is 10 in. by 5 in. on the floor plan is 20 ft by 10 ft.

30c.

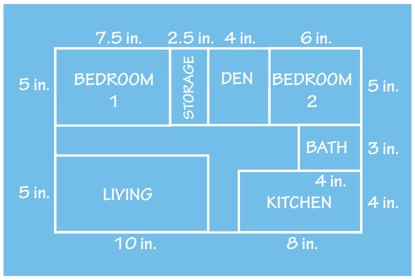
Bedroom 1:	15 ft by 10 ft
Storage:	5 ft by 10 ft
Den:	8 ft by 10 ft
Bedroom 2:	12 ft by 10 ft
Bath:	8 ft by 6 ft
Kitchen:	16 ft by 8 ft
Living:	20 ft by 10 ft

30. ★ **EXTENDED RESPONSE** The figure below shows an architect's initial layout of the floor plan for a house. The scale is $\frac{1}{2}$ inch = 1 foot. **See margin.**

a. **Interpret** How is a dilation of the floor plan used in the house construction? What is the scale factor of the dilation?

b. **Model** Explain how to find the actual size of the living room in the house using the floor plan.

c. **Calculate** Find the actual size of each room in the house.

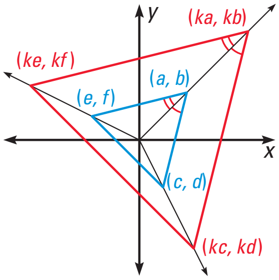


31. ★ **OPEN-ENDED** A transformation in the coordinate plane is described using the notation $(x, y) \to (2x, 5y)$. *Explain* why the transformation is not a similarity transformation by showing how it affects the angles and sides of a polygon. **See margin.**

C 32. **CHALLENGE** A dilation of a triangle is shown, in which the center of dilation lies in the interior of the triangle. **See margin.**

a. Use the slope formula to show that corresponding sides are parallel.

b. The black rays in the diagram are transversals that intersect parallel segments. Prove the angles marked are congruent. *Explain* why the angles of the preimage and image triangles are preserved under the dilation.

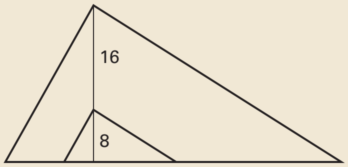


5 ASSESS AND RETEACH

Daily Homework Quiz

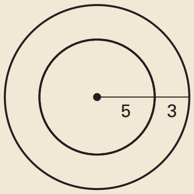
Describe the dilation that moves the smaller figure onto the larger figure.

1.



dilation with scale factor 3 and center at the intersection of the thin lines

2.



dilation with scale factor $\frac{8}{5}$ and center at center of the circles

3. The coordinates of the vertices of a $\triangle ABC$ and its image $\triangle DEF$ are given. Describe the transformation(s) that move $\triangle ABC$ figure onto $\triangle DEF$.

$A(1, 1)$, $B(-2, 2)$, $C(2, 2)$;
$D(2, -2)$, $E(-4, -4)$, $F(4, -4)$

Sample answer: **dilation with center (0, 0) and scale factor 2, then reflection in the *x*-axis**

Diagnosis/Remediation

- Practice B in Chapter Resources
- Study Guide in Chapter Resources

Challenge

Additional challenge is available in the Chapter Resources.

32b. The black ray that passes through (a, b) and (ka, kb) is a transversal intersecting parallel segments, so the angles marked are congruent. Therefore, their sums are also equal. Similar reasoning applies to the other angles of the triangle. So, angles are preserved under the dilation.

31. *Sample answer:* An isosceles right triangle with vertices (0, 0), (1, 0), and (1, 1), is mapped to a triangle with vertices (0, 0), (2, 0), and (2, 5). The image triangle is not isosceles; its legs are not the same length and its acute angles do not have the same measure. Image side lengths are not in proportion to preimage side lengths, and angles are not preserved, so the transformation is not a similarity transformation.

32a. The slope of an image side is the same as the slope of a corresponding preimage side. For example:

$$\frac{kd - kb}{kc - ka} = \frac{k(d - b)}{k(c - a)} = \frac{(d - b)}{(c - a)}$$

Similar reasoning applies to the other sides of the triangle.

1 PLAN AND PREPARE

Explore the Concept

- Students will build similar triangles given two angle measures.
- This activity supplements the study of the Angle-Angle (AA) Similarity Postulate.

Materials

Each student will need:

- ruler
- protractor

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Key Discovery

If two triangles have two pairs of corresponding angles congruent, then a dilation (or a combination of a dilation and a rigid motion) can be used to move one triangle onto the other.

3 ASSESS AND RETEACH

1. In the Explore, suppose you drew $\triangle GHJ$ inside $\triangle DEF$ so no sides were parallel to sides of $\triangle DEF$. Would you be able to find a center of dilation? ***Sample answer:* No; preimage and image sides in a dilation must be parallel, so you would not be able to find a center of dilation.**

2. In the Explore, suppose you drew $\triangle GHJ$ outside $\triangle DEF$. Would you be able to find a center of dilation? ***Sample answer:* Yes; draw lines through corresponding vertices to find the center of dilation; it would lie outside the triangles.**

Investigating Geometry **ACTIVITY** *Use before Prove Triangles Similar by AA*

Dilations and AA Similarity

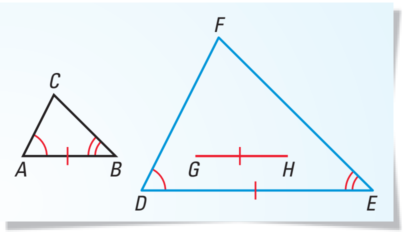
MATERIALS • ruler • protractor

QUESTION **How can you use a dilation to map a triangle onto a similar triangle with two pairs of corresponding angles congruent?**

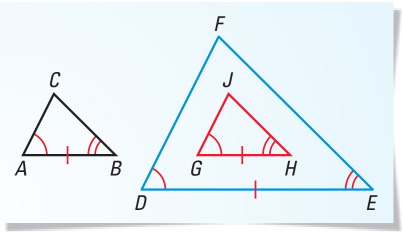
Recall that a dilation preserves angle measures but not lengths.

EXPLORE **Build similar triangles given two angles**

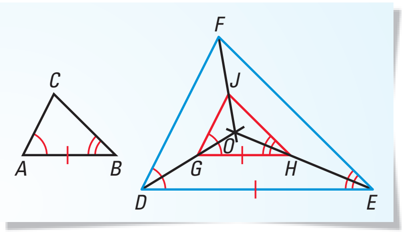
STEP 1 ***Draw triangles*** Draw $\triangle ABC$ with any angles. Use a protractor to draw a larger $\triangle DEF$, with $\angle D \cong \angle A$ and $\angle E \cong \angle B$. Inside $\triangle DEF$, draw a segment parallel to $\overline{DE}$ the same length as $\overline{AB}$. Label its endpoints G and H.



STEP 2 ***Copy angles*** Copy $\angle D$ at G and copy $\angle E$ at H. Extend the sides until they intersect. Label the intersection J.



STEP 3 ***Find center of dilation*** Draw three rays that intersect: $\overrightarrow{DG}$, $\overrightarrow{EH}$, and $\overrightarrow{FJ}$. Label their point of intersection O.



DRAW CONCLUSIONS **Use your observations to complete these exercises** **1–5. See margin.**

1. Measure $\angle C$, $\angle F$, and $\angle J$. What do you notice?
2. A dilation maps $\triangle DEF$ to $\triangle GHJ$. Why is $\angle D \cong \angle G$ and $\angle E \cong \angle H$?
3. Find $\frac{GO}{DO}$, $\frac{HO}{EO}$, and $\frac{JO}{FO}$. What is the scale factor of the dilation?
4. Prove that $\triangle GHJ \cong \triangle ABC$. *Justify* your reasoning.
5. What combination of transformations maps $\triangle DEF$ to $\triangle ABC$?

1. $m\angle C = m\angle F = m\angle J$; the third angles of the triangles are congruent because the sum of the angles of a triangle is 180°.

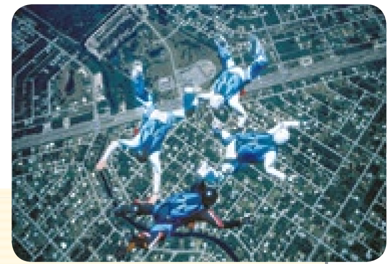
2. Dilations preserve angle measures, so corresponding angles of $\triangle DEF$ and $\triangle GHJ$ are congruent.

3. Check students' work. The ratios should be equal to the scale factor of the dilation.

4. $\angle A \cong \angle G$ and $\angle B \cong \angle H$ because dilation preserves angle measures, and $\overline{AB} \cong \overline{GH}$ because this was given in Step 1. So, $\triangle GHJ \cong \triangle ABC$ by ASA.

5. dilation with scale factor $\frac{GO}{DO}$ and center O, then translation a distance GA

6.3 Prove Triangles Similar by AA



Before You used the AAS Congruence Theorem.

Now You will use the AA Similarity Postulate.

Why? So you can use similar triangles to understand aerial photography, as in Ex. 34.

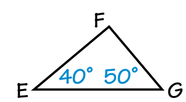
Key Vocabulary
- similar polygons

ACTIVITY ANGLES AND SIMILAR TRIANGLES

QUESTION What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?

Materials:
- protractor
- metric ruler

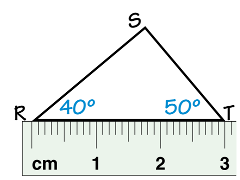
STEP 1 **Draw** $\triangle EFG$ so that $m\angle E = 40°$ and $m\angle G = 50°$.



STEP 2 **Draw** $\triangle RST$ so that $m\angle R = 40°$ and $m\angle T = 50°$, and $\triangle RST$ is not congruent to $\triangle EFG$.

STEP 3 **Calculate** $m\angle F$ and $m\angle S$ using the Triangle Sum Theorem. Use a protractor to check that your results are true. **90°, 90°**

STEP 4 **Measure** and record the side lengths of both triangles. Use a metric ruler.



DRAW CONCLUSIONS

1. Are the triangles similar? Explain your reasoning. **See margin.**
2. Repeat the steps above using different angle measures. Make a conjecture about two triangles with two pairs of congruent corresponding angles. **See margin.**

1. Yes; the corresponding angles are congruent, and the corresponding side lengths are proportional.

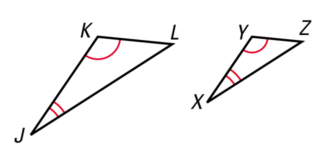
2. Two triangles with two pairs of congruent corresponding angles are similar triangles.

TRIANGLE SIMILARITY The Activity suggests that two triangles are similar if two pairs of corresponding angles are congruent. In other words, you do not need to know the measures of the sides or the third pair of angles.

POSTULATE *For Your Notebook*

POSTULATE 22 Angle-Angle (AA) Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



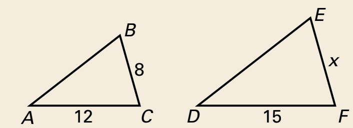
$\triangle JKL \sim \triangle XYZ$

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. In $\triangle ABC$ and $\triangle XZW$, $m\angle A = m\angle X$ and $m\angle B = m\angle Z$. What can you conclude about $m\angle C$ and $m\angle W$? **They are the same.**
2. Solve $\frac{x}{18} = \frac{54}{9}$. **108**
3. $\triangle ABC \sim \triangle DEF$. Find x. **10**



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with previous lesson
- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How can you show that two triangles are similar? **Tell students they will learn how to answer this question by showing that two pairs of angles are congruent.**

Motivating the Lesson

Ask students how to describe the angles in two similar figures. Then ask them how they could check if two triangles in a design or graphic drawing are similar. They should be able to use the Triangle Sum theorem to conclude that they need to check for just two pairs of congruent angles.

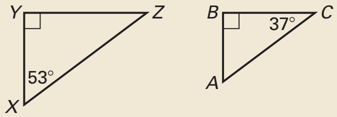
3 TEACH

Activity Note

The purpose of this activity is to show that if two angles in two triangles have the same angle measure, then the third angles must have the same measure and the corresponding sides are proportional.

Extra Example 1

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

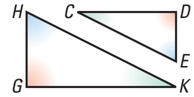


$\angle Y \cong \angle B$ because both are right angles. By the Triangle Sum Theorem, $m\angle Z = 37°$ so $m\angle Z \cong \angle C$. $\triangle XYZ \sim \triangle ABC$ by the AA Similarity Postulate.

Key Question to Ask for Example 1

- What is the name of the triangle that is similar to $\triangle EDC$? How do you know the order for the vertices? **$\triangle EDC \sim \triangle HGK$; corresponding vertices appear in the same position for each triangle.**

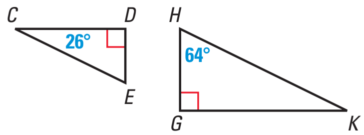
DRAW DIAGRAMS



Use colored pencils to show congruent angles. This will help you write similarity statements.

EXAMPLE 1 Use the AA Similarity Postulate

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



Solution

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

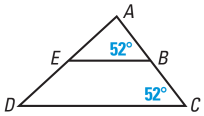
By the Triangle Sum Theorem, $26° + 90° + m\angle E = 180°$, so $m\angle E = 64°$. Therefore, $\angle E$ and $\angle H$ are congruent.

▶ So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Postulate.

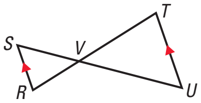
EXAMPLE 2 Show that triangles are similar

Show that the two triangles are similar.

a. $\triangle ABE$ and $\triangle ACD$



b. $\triangle SVR$ and $\triangle UVT$



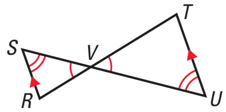
Solution

a. You may find it helpful to redraw the triangles separately.

Because $m\angle ABE$ and $m\angle C$ both equal 52°, $\angle ABE \cong \angle C$. By the Reflexive Property, $\angle A \cong \angle A$.

▶ So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Postulate.

b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem. The diagram shows $\overline{RS} \parallel \overline{UT}$ so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem.

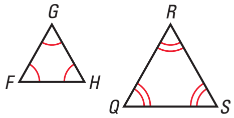


▶ So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Postulate.

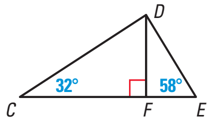
✓ GUIDED PRACTICE for Examples 1 and 2

Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



3. REASONING Suppose in Example 2, part (b), $\overline{SR} \nparallel \overline{TU}$. Could the triangles still be similar? *Explain.*
Yes; if $\angle S \cong \angle T$, the triangles are similar by the AA Similarity Postulate.

1. In each triangle all three angles measure 60°, so by the AA Similarity Postulate the triangles are similar; $\triangle FGH \sim \triangle QRS$.

2. Since $m\angle CDF = 58°$ by the Triangle Sum Theorem and $m\angle DFE = 90°$ by the Linear Pair Postulate the two triangles are similar by the AA Similarity Postulate; $\triangle CDF \sim \triangle DEF$.

Differentiated Instruction

Below Level Ask students to draw two scalene triangles. Then have them draw a segment connecting two sides of each triangle, parallel to the third side in one triangle and not parallel in the other triangle. Ask the students to make a conjecture about which drawing has similar triangles, then have them use protractors and rulers to verify their conjecture.

See also the *Differentiated Instruction Resources* for more strategies.

INDIRECT MEASUREMENT Previously, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 Standardized Test Practice

A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is five feet four inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

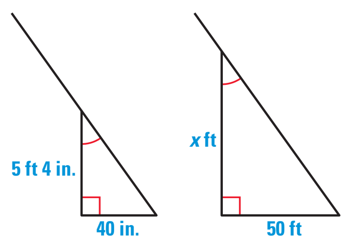
Ⓐ 12 feet Ⓑ 40 feet

Ⓒ 80 feet Ⓓ 140 feet

ELIMINATE CHOICES
Notice that the woman's height is greater than her shadow's length. So the flagpole must be taller than its shadow's length. Eliminate choices A and B.

Solution

The flagpole and the woman form sides of two right triangles with the ground, as shown below. The sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.



You can use a proportion to find the height x. Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$ Write proportion of side lengths.

$40x = 64(50)$ Cross Products Property

$x = 80$ Solve for x.

▶ The flagpole is 80 feet tall. The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Example 3

4. **WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow? **36.25 in.**

5. You are standing in your backyard, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.
Sample answer: $\frac{\text{tree height}}{\text{your height}} = \frac{\text{length of tree shadow}}{\text{length of your shadow}}$

Differentiated Instruction

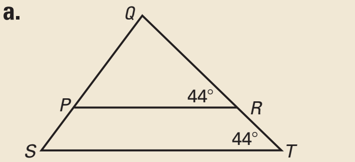
Kinesthetic Learners After discussing **Example 3**, take students outside on a sunny day to find and solve a problem similar to **Example 3**. Point out that they can use this strategy to find the height of any tall object, such as a tree or a building. Have students draw a diagram showing the two triangles. You may wish to have different groups of students use people of different heights to find the height of the same object and then compare their answers.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 2

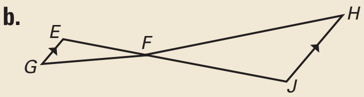
Show that the two triangles are similar.

a.



$m\angle QRP = 44°$ and $m\angle QTS = 44°$, so $\angle QRP \cong \angle QTS$. Also, $\angle Q \cong \angle Q$. So $\triangle QRP \sim \triangle QTS$ by the AA Similarity Postulate.

b.



$\angle EFG \cong \angle JFH$ because they are vertical angles, and $\overline{EG} \parallel \overline{JH}$ so $\angle E \cong \angle J$ by the Alternate Interior Angles Theorem. So $\triangle EFG \sim \triangle JFH$ by the AA Similarity Postulate.

Study Strategy

For Example 2, instruct students to redraw $\triangle AEB$ and $\triangle ADC$ separately and mark the congruent parts.

Extra Example 3

A school building casts a shadow that is 26 feet long. At the same time a student standing nearby, who is 71 inches tall, casts a shadow that is 48 inches long. How tall is the building to the nearest foot? **C**

Ⓐ 18 ft Ⓑ 33 ft

Ⓒ 38 ft Ⓓ 131 ft

Teaching Strategy

Point out that the procedure used in Example 3 to find the height of an object indirectly is useful for vertical objects like buildings and trees. Encourage them to draw a diagram and label it carefully.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you show that two triangles are similar?

- **If two pairs of angles of two triangles are congruent, you can use the AA Similarity Postulate to conclude that the triangles are similar.**

Show that two pairs of angles are congruent and apply the AA Similarity Postulate.

6.3 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS** Exs. 9, 13, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 18, 19, 20, 33, and 38

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: EP for 3.1 Exs. 7–11
Exs. 1–20, 31–35

Average:
Day 1:
Exs. 1, 2–12 even, 15–25, 31–37

Advanced:
Day 1:
Exs. 1, 2, 3–7 odd, 11–14, 16–30*, 32, 34–40*

Block:
Exs. 1, 2–12 even, 15–25, 31–37 (with previous lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 6, 12, 18, 31, 34

Average: 10, 12, 22, 32, 35

Advanced: 11, 16, 24, 32, 36

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are __?__. **similar**

2. ★ **WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? *Explain.* **No; the ratio of corresponding sides would be the same but they would not necessarily be congruent.**

EXAMPLE 1 for Exs. 3–11

REASONING **Use the diagram to complete the statement.**

3. $\triangle ABC \sim$ __?__ $\triangle FED$

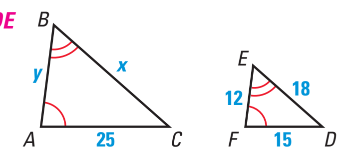
4. $\frac{BA}{?} = \frac{AC}{?} = \frac{CB}{?}$ **EF, FD, DE**

5. $\frac{25}{?} = \frac{?}{12}$ **15, y**

6. $\frac{?}{25} = \frac{18}{?}$ **15, x**

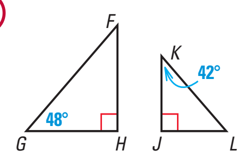
7. $y =$ __?__ **20**

8. $x =$ __?__ **30**



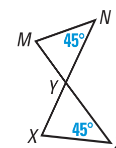
AA SIMILARITY POSTULATE **In Exercises 9–14, determine whether the triangles are similar. If they are, write a similarity statement.**

9.



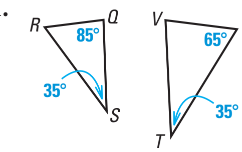
similar; △FGH ~ △KLJ

10.



similar; △NYM ~ △ZYX

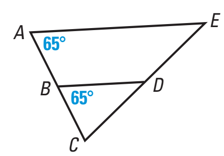
11.



not similar

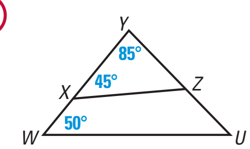
EXAMPLE 2 for Exs. 12–16

12.



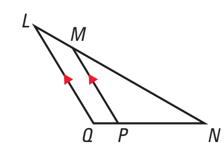
similar; △CBD ~ △CAE

13.



similar; △YZX ~ △YWU

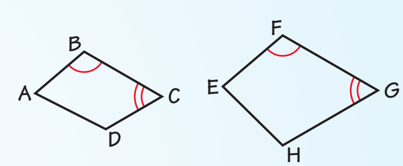
14.



similar; △NMP ~ △NLQ

15. **ERROR ANALYSIS** *Explain* why the student's similarity statement is incorrect.

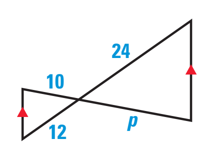
ABCD ~ EFGH
by AA Similarity
Postulate



The AA Similarity Postulate is for triangles, not quadrilaterals.

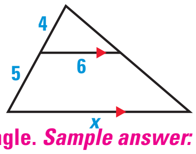
16. ★ **MULTIPLE CHOICE** What is the value of p? **B**

   Ⓐ 5 Ⓑ 20
   Ⓒ 28.8 Ⓓ Cannot be determined



B

17. ERROR ANALYSIS A student uses the proportion $\frac{4}{6} = \frac{5}{x}$ to find the value of x in the figure. *Explain* why this proportion is incorrect and write a correct proportion. **5 should be replaced by 9, which is the length of the corresponding side of the larger triangle.** ***Sample answer:*** $\frac{4}{6} = \frac{9}{x}$.



★ **OPEN-ENDED MATH In Exercises 18 and 19, make a sketch that can be used to show that the statement is false.** **18, 19. See margin.**

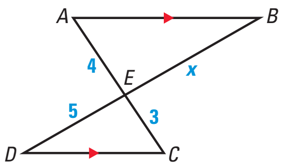
18. If two pairs of sides of two triangles are congruent, then the triangles are similar.

19. If the ratios of two pairs of sides of two triangles are proportional, then the triangles are similar.

20. ★ **MULTIPLE CHOICE** In the figure at the right, find the length of $\overline{BD}$. **A**

Ⓐ $\frac{35}{3}$ Ⓑ $\frac{37}{5}$

Ⓒ $\frac{20}{3}$ Ⓓ $\frac{12}{5}$



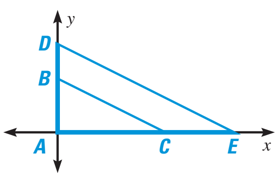
ALGEBRA Find coordinates for point *E* so that $\triangle ABC \sim \triangle ADE$.

21. $A(0, 0)$, $B(0, 4)$, $C(8, 0)$, $D(0, 5)$, $E(x, y)$ **(10, 0)**

22. $A(0, 0)$, $B(0, 3)$, $C(4, 0)$, $D(0, 7)$, $E(x, y)$ $\left(\frac{28}{3}, 0\right)$

23. $A(0, 0)$, $B(0, 1)$, $C(6, 0)$, $D(0, 4)$, $E(x, y)$ **(24, 0)**

24. $A(0, 0)$, $B(0, 6)$, $C(3, 0)$, $D(0, 9)$, $E(x, y)$ $\left(\frac{9}{2}, 0\right)$



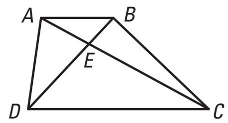
25. MULTI-STEP PROBLEM In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $AE = 6$, $AB = 8$, $CE = 15$, and $DE = 10$.

a. Copy the diagram and mark all given information. **See margin.**

b. List two pairs of congruent angles in the diagram.

c. Name a pair of similar triangles and write a similarity statement. **$\triangle ABE$ and $\triangle CDE$, $\triangle ABE \sim \triangle CDE$**

d. Find BE and DC. **4, 20**



25b. *Sample answer:* $\angle ABE$ and $\angle CDE$, $\angle BAE$ and $\angle DCE$

C

REASONING In Exercises 26–29, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? *Explain* why or why not.

26. $m\angle J = 71°$, $m\angle K = 52°$, $m\angle X = 71°$, and $m\angle Z = 57°$
Yes; in $\triangle JKL$ $m\angle L = 57°$ making the triangles similar by the AA Similarity Postulate.

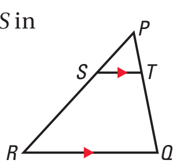
27. $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150°$.
Yes; either $m\angle X$ or $m\angle Y$ could be 90°, and the other angles could be the same.

28. $m\angle J = 87°$ and $m\angle Y = 94°$
No; 87° + 94° = 181° is already greater then the possible total for three angles in a triangle.

29. $m\angle J + m\angle K = 85°$ and $m\angle Y + m\angle Z = 80°$

29. No; since $m\angle J + m\angle K = 85°$ then $m\angle L = 95°$. Since $m\angle Y + m\angle Z = 80°$ then $m\angle X = 100°$ and thus neither $\angle Y$ nor $\angle Z$ can measure 95°.

30. CHALLENGE If $PT = x$, $PQ = 3x$, and $SR = \frac{8}{3}x$, find PS in terms of x. *Explain* your reasoning.
$\frac{4}{3}x$; solve the proportion $\frac{a}{a + \frac{8}{3}x} = \frac{x}{3x}$ where $PS = a$.



Avoiding Common Errors

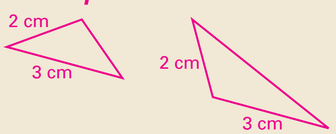
Exercise 13 Students may assume that if $\overline{XZ}$ is not parallel to $\overline{WU}$ then the triangles are not similar. Caution them to find the measures of the third pair of angles before answering the question.

Study Strategy

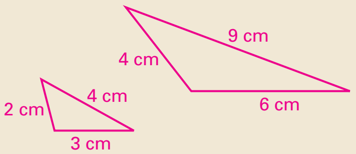
Exercise 25 Encourage students to separate the triangles to help them see which pairs of angle of triangles are similar.

Exercises 26–29 Instruct students to look for counterexamples by drawing triangles for each of these exercises.

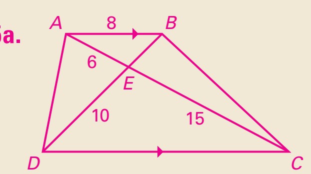
18. *Sample:*



19. *Sample:*



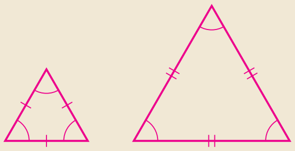
25a.



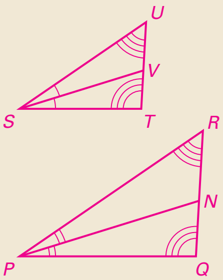
Mathematical Reasoning

Exercise 35 This exercise is a proof that if two triangles are similar, then the angle bisectors of corresponding angles have the same scale factor as the corresponding sides.

33.

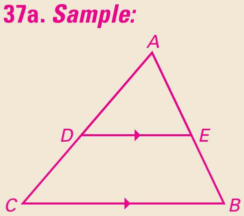


35.



Since $\triangle STU \sim \triangle PQR$ you know that $\angle T \cong \angle Q$ and $\angle UST \cong \angle RPQ$. Since $\overline{SV}$ bisects $\angle TSU$ and $\overline{PN}$ bisects $\angle QPR$ you know that $\angle USV \cong \angle VST$ and $\angle RPN \cong \angle NPQ$ by definition of angle bisector. You know that $m\angle USV + m\angle VST = m\angle UST$ and $m\angle RPN + m\angle NPQ = m\angle RPQ$, therefore $2m\angle VST = 2m\angle NPQ$ using the Substitution Property of Equality. You now have $\angle VST \cong \angle NPQ$, which makes $\triangle VST \sim \triangle NPQ$ using the AA Similarity Postulate. From this you know that $\frac{SV}{PN} = \frac{ST}{PQ}$.

37a. *Sample:*

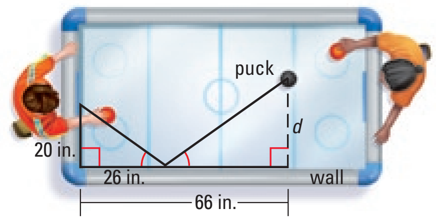


37e. The measures of the angles change, but the equalities remain the same. The lengths of the sides change, but they remain proportional; yes; the triangles remain similar by the AA Similarity Postulate.

PROBLEM SOLVING

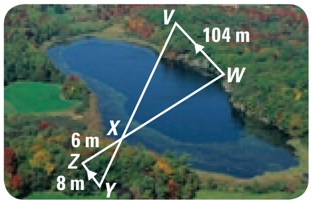
EXAMPLE 3 A
for Exs. 31–32

31. AIR HOCKEY An air hockey player returns the puck to his opponent by bouncing the puck off the wall of the table as shown. From physics, the angles that the path of the puck makes with the wall are congruent. What is the distance d between the puck and the wall when the opponent returns it? **about 30.8 in.**



32a. Angle-Angle Similarity Postulate

32. LAKES You can measure the width of the lake using a surveying technique, as shown in the diagram.



a. What postulate or theorem can you use to show that the triangles are similar?

b. Find the width of the lake, WX. **78 m**

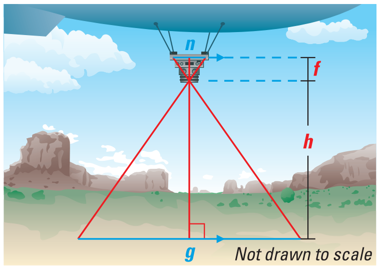
c. If $XY = 10$ meters, find VX. **130 m**

33. The measure of all angles in an equilateral triangle is 60°; see margin for art.

33. ★ **SHORT RESPONSE** *Explain* why all equilateral triangles are similar. Include sketches in your answer.

B

34. AERIAL PHOTOGRAPHY Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance g of 50 meters. Use the proportion $\frac{f}{h} = \frac{n}{g}$ to estimate the altitude h that the blimp should fly at to take the photo. In the proportion, use $f = 8$ centimeters and $n = 3$ centimeters. These two variables are determined by the type of camera used. **$133\frac{1}{3}$ m**



35. PROOF Use the given information to draw a sketch. Then write a proof. **See margin.**

GIVEN ▶ $\triangle STU \sim \triangle PQR$
Point V lies on $\overline{TU}$ so that $\overline{SV}$ bisects $\angle TSU$.
Point N lies on $\overline{QR}$ so that $\overline{PN}$ bisects $\angle QPR$.

PROVE ▶ $\frac{SV}{PN} = \frac{ST}{PQ}$

36. PROOF Prove that if an acute angle in one right triangle is congruent to an acute angle in another right triangle, then the triangles are similar.
***Sample answer:* If $\angle C$ in $\triangle ABC$ and $\angle F$ in $\triangle DEF$ are right angles, then $\angle C \cong \angle F$. If also $\angle A \cong \angle D$, then $\triangle ABC \sim \triangle DEF$ by the AA Similarity Postulate.**

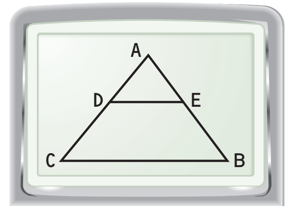
○ = See **WORKED-OUT SOLUTIONS** in Student Resources
★ = **STANDARDIZED TEST PRACTICE**

39. Let $\triangle ABC \sim \triangle DEF$, let $\overline{AN}$ bisect $\angle BAC$, and let $\overline{DM}$ bisect $\angle EDF$. By the definition of similar triangles, $\angle B \cong \angle E$ and $\angle BAC \cong \angle EDF$. By the definition of angle bisector, $\angle BAN \cong \angle NAC$ and $\angle EDM \cong \angle MDF$. The Angle Addition Postulate gives $m\angle BAN + m\angle NAC = m\angle BAC$ and $m\angle EDM + m\angle MDF = m\angle EDF$. By substitution we get $m\angle BAN + m\angle NAC = m\angle EDM + m\angle MDF$, and then $m\angle BAN + m\angle BAN = m\angle EDM + m\angle EDM$, and then $2m\angle BAN = 2m\angle EDM$, so $m\angle BAN = m\angle EDM$. Now, by the AA Similarity Postulate, $\triangle BAN \sim \triangle EDM$ so $\frac{AN}{DM} = \frac{AB}{DE}$ where $\frac{AB}{DE}$ is the scale factor.

37b.
$m\angle ADE = m\angle ACB$ and $m\angle AED = m\angle ABC$

40. The two right triangles formed by the altitudes and the two sides measuring a and b are similar by the AA Similarity Postulate. Since the ratio of the hypotenuses is $\frac{b}{a}$ then the ratio of corresponding sides, which are the altitudes of the original triangles is the same ratio by Corresponding Lengths in similar Polygons.

37. TECHNOLOGY Use a graphing calculator or computer.



a. Draw $\triangle ABC$. Draw $\overline{DE}$ through two sides of the triangle, parallel to the third side. **See margin.**

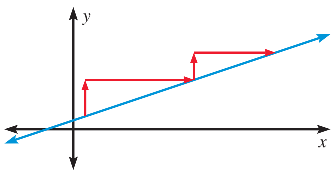
b. Measure $\angle ADE$ and $\angle ACB$. Measure $\angle AED$ and $\angle ABC$. What do you notice?

c. What does a postulate in this lesson tell you about $\triangle ADE$ and $\triangle ACB$? $\triangle ADE \sim \triangle ACB$

d. Measure all the sides. Show that corresponding side lengths are proportional. *Sample answer:* $\frac{AD}{AC} = \frac{AE}{AB} = \frac{DE}{CB} = \frac{1}{2}$

e. Move vertex A to form new triangles. How do your measurements in parts (b) and (d) change? Are the new triangles still similar? *Explain.* **See margin.**

C **38. ★ EXTENDED RESPONSE** *Explain* how you could use similar triangles to show that any two points on a line can be used to calculate its slope.



Since the two triangles are similar the ratio of corresponding sides are the same therefore compare the vertical rise to the horizontal run.

39. CORRESPONDING LENGTHS Without using the Corresponding Lengths Property, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor. **See margin.**

40. CHALLENGE Prove that if the lengths of two sides of a triangle are a and b respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

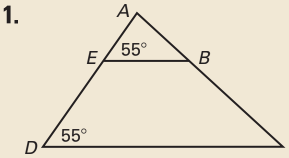
5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

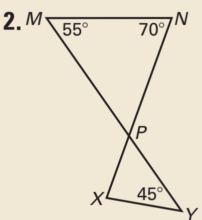
Determine if the two triangles are similar. If they are, write a similarity statement.

1.

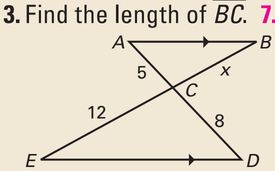


yes; $\triangle ABE \sim \triangle ACD$

2. **no**



3. Find the length of $\overline{BC}$. **7.5**



4. A tree casts a shadow that is 30 feet long. At the same time a person standing nearby, who is five feet two inches tall, casts a shadow that is 50 inches long. How tall is the tree to the nearest foot? **37 ft**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Determine whether the two triangles are similar.

1. $\triangle ABC$: $m\angle A = 90°$, $m\angle B = 44°$; $\triangle DEF$: $m\angle D = 90°$, $m\angle E = 46°$ **similar**
2. $\triangle ABC$: $m\angle A = 132°$, $m\angle B = 24°$; $\triangle DEF$: $m\angle D = 90°$, $m\angle F = 24°$ **not similar**
3. Solve $\frac{6}{12} = \frac{x-1}{8}$. **5**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

• See *Teaching Guide/Lesson Plan*.

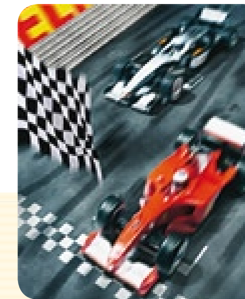
2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How do you prove that two triangles are similar by using the SSS Similarity Theorem? **Tell students they will learn how to answer this question by using the ratios of pairs of sides.**

6.4 Prove Triangles Similar by SSS and SAS



Before You used the AA Similarity Postulate to prove triangles similar.

Now You will use the SSS and SAS Similarity Theorems.

Why? So you can show that triangles are similar, as in Ex. 28.

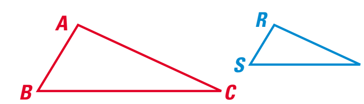
Key Vocabulary
- ratio
- proportion
- similar polygons

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

THEOREM *For Your Notebook*

THEOREM 6.2 Side-Side-Side (SSS) Similarity Theorem

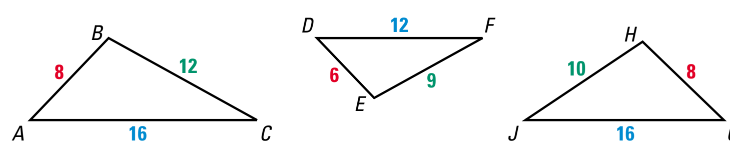
If the corresponding side lengths of two triangles are proportional, then the triangles are similar.



If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

EXAMPLE 1 Use the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

APPLY THEOREMS
When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides $\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$

Longest sides $\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$

Remaining sides $\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$

▶ All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

Shortest sides $\frac{AB}{GH} = \frac{8}{8} = 1$

Longest sides $\frac{CA}{JG} = \frac{16}{16} = 1$

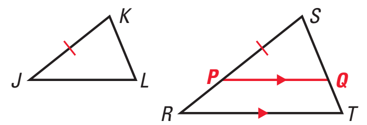
Remaining sides $\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$

▶ The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF SSS Similarity Theorem

GIVEN ▶ $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

PROVE ▶ $\triangle RST \sim \triangle JKL$



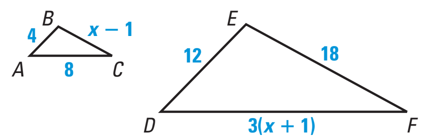
USE AN AUXILIARY LINE
The Parallel Postulate allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point P parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate P on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$.

You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Postulate, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2 Use the SSS Similarity Theorem

ALGEBRA **Find the value of x that makes $\triangle ABC \sim \triangle DEF$.**



Solution

STEP 1 **Find** the value of x that makes corresponding side lengths proportional.

CHOOSE A METHOD
You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

$\frac{4}{12} = \frac{x-1}{18}$	**Write proportion.**
$4 \cdot 18 = 12(x - 1)$	**Cross Products Property**
$72 = 12x - 12$	**Simplify.**
$7 = x$	**Solve for *x*.**

STEP 2 **Check** that the side lengths are proportional when $x = 7$.

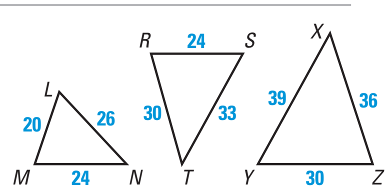
$BC = x - 1 = 6$ $\qquad$ $DF = 3(x + 1) = 24$

$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{12} = \frac{6}{18}$ ✓ $\qquad$ $\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \rightarrow \frac{4}{12} = \frac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Which of the three triangles are similar? Write a similarity statement. **$\triangle MLN \sim \triangle ZYX$**
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle. **15, 16.5**



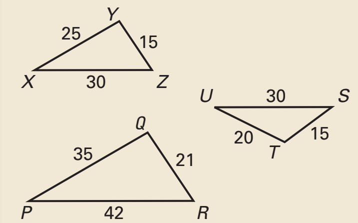
Motivating the Lesson

Ask students to draw a triangle and measure each side. Then have them draw a similar triangle with sides half as long. Discuss how they know that the smaller triangle is similar to the larger one.

3 TEACH

Extra Example 1

Is either $\triangle PQR$ or $\triangle STU$ similar to $\triangle XYZ$?



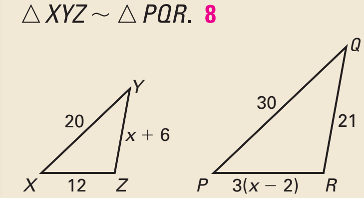
$\triangle XYZ \sim \triangle PQR$; $\triangle XYZ$ is not similar to $\triangle STU$.

Key Question to Ask for Example 1

- Do the ratios of all three pairs of corresponding sides have to be equal if the two triangles are similar? **yes**

Extra Example 2

Find the value of x that makes $\triangle XYZ \sim \triangle PQR$. **8**

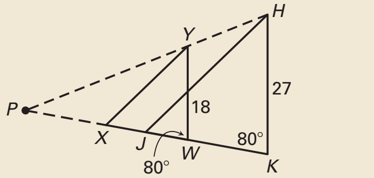


Key Questions to Ask for Example 2

- What is the scale factor for the triangles? **4 : 12 or 1 : 3**
- How is the scale factor used to find x? **You write and solve a proportion involving the scale factor and x.**

Extra Example 3

You enlarge $\triangle XYW$ to $\triangle JHK$ as shown. Is $\triangle XYW$ similar to $\triangle JHK$?



$XW = 16$, $JK = 24$

yes

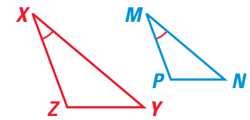
Key Questions to Ask for Example 3

- What method can you use to show that the triangles are similar? **You can show that a corresponding pair of angles are congruent and that the lengths of the sides that include the angles are proportional.**
- Are the lengths of all three pairs of sides proportional? **yes**

THEOREM *For Your Notebook*

THEOREM 6.3 Side-Angle-Side (SAS) Similarity Theorem

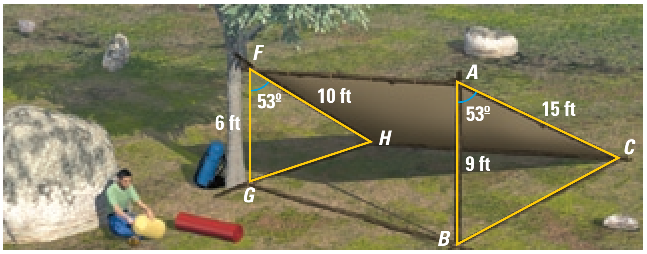
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.



If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

EXAMPLE 3 Use the SAS Similarity Theorem

LEAN-TO SHELTER You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



Solution

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides $\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$ **Longer sides** $\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$

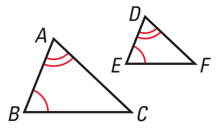
The lengths of the sides that include $\angle A$ and $\angle F$ are proportional.

▶ So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$. Yes, you can make the right end similar to the left end of the shelter.

CONCEPT SUMMARY *For Your Notebook*

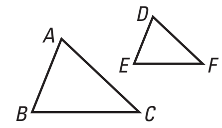
Triangle Similarity Postulate and Theorems

AA Similarity Postulate



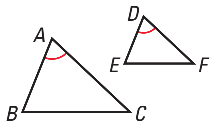
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem

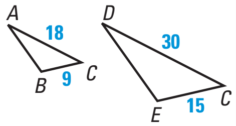


If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

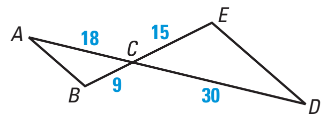
EXAMPLE 4 Choose a method

VISUAL REASONING

To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$ **Longer sides** $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

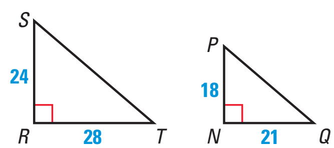
The corresponding side lengths are proportional. The included angles $\angle ACB$ and $\angle DCE$ are congruent because they are vertical angles. So, $\triangle ACB \sim \triangle DCE$ by the SAS Similarity Theorem.

Animated Geometry at my.hrw.com

GUIDED PRACTICE for Examples 3 and 4

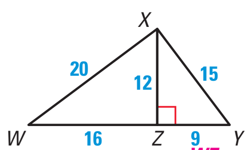
Explain **how to show that the indicated triangles are similar.**

3. $\triangle SRT \sim \triangle PNQ$



$\angle R \cong \angle N$ and $\frac{SR}{PN} = \frac{RT}{NQ} = \frac{4}{3}$ therefore the triangles are similar by the SAS Similarity Theorem.

4. $\triangle XZW \sim \triangle YZX$



$\angle WZX \cong \angle XZY$ and $\frac{WZ}{XZ} = \frac{XZ}{YZ} = \frac{WX}{XY} = \frac{4}{3}$ therefore the triangles are similar by either SSS or SAS Similarity Theorems.

6.4 EXERCISES

HOMEWORK KEY

○ = **SEE WORKED-OUT SOLUTIONS** for Exs. 3, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 32, 34, and 36

SKILL PRACTICE

A

1. VOCABULARY You plan to prove that $\triangle ACB$ is similar to $\triangle PXQ$ by the SSS Similarity Theorem. Copy and complete the proportion that is needed to use this theorem: $\frac{AC}{?} = \frac{?}{XQ} = \frac{AB}{?}$.

PX CB PQ

2. ★ WRITING If you know two triangles are similar by the SAS Similarity Theorem, what additional piece(s) of information would you need to know to show that the triangles are congruent?

You would need to know that one pair of corresponding sides is congruent.

EXAMPLES 1 and 2 for Exs. 3–6

SSS SIMILARITY THEOREM **Verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.**

3. $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

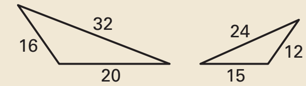
$\frac{18}{12} = \frac{15}{10} = \frac{12}{8}$, $\frac{3}{2}$

4. $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
$\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$

$\frac{10}{25} = \frac{16}{40} = \frac{20}{50}$, $\frac{2}{5}$

Extra Example 4

Tell what method you would use to show that the triangles are similar.



SSS Similarity Theorem

Teaching Strategy

Ask students to summarize the triangle similarity postulates and theorems by drawing and labeling a diagram for each and writing a conditional statement for each situation.

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 4**. This activity is also part of **Power Presentations**.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you prove that two triangles are similar using the SSS Similarity Theorem?

- You can prove triangles are similar by the AA Similarity Postulate, the SSS Similarity Theorem, and the SAS Similarity Theorem.

Compare the ratio of each pair of corresponding side lengths. If they are all equal, then the triangles are similar by the SSS Similarity Theorem.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–6, 28–30, 32
Day 2:
Exs. 7–17, 31, 33, 34

Average:
Day 1:
Exs. 1–6, 18–23, 28–30, 32
Day 2:
Exs. 7–17, 24, 31, 33–37

Advanced:
Day 1:
Exs. 1, 2, 5, 6, 18–23, 25–30*, 32
Day 2:
Exs. 8, 9, 11, 12, 14–17, 24, 31, 33–38*

Block:
Exs. 1–24, 28–37

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 8, 10, 29, 33
Average: 5, 8, 11, 30, 33
Advanced: 6, 9, 12, 32, 33

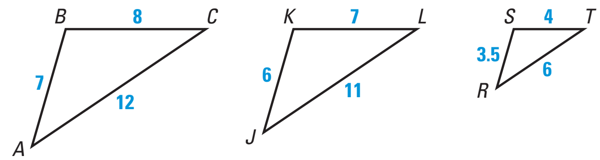
Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

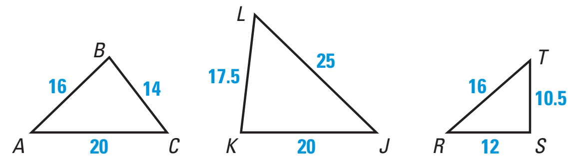
Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

5. SSS SIMILARITY THEOREM Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$? ***$\triangle RST$***



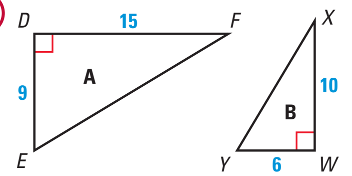
6. SSS SIMILARITY THEOREM Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$? ***$\triangle JKL$***



EXAMPLE 3 for Exs. 7–9

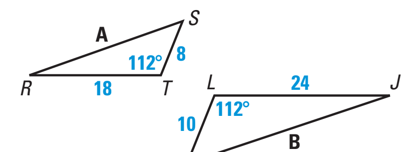
SAS SIMILARITY THEOREM Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.

7.



similar; $\triangle FDE \sim \triangle XWY$; 2 : 3

8.



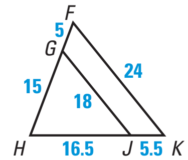
not similar

9. *xy* **ALGEBRA** Find the value of n that makes $\triangle PQR \sim \triangle XYZ$ when $PQ = 4$, $QR = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle Q \cong \angle Y$. Include a sketch. ***3; see margin for art.***

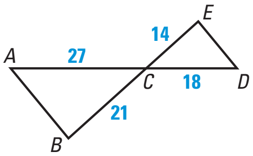
EXAMPLE 4 for Exs. 10–12

SHOWING SIMILARITY Show that the triangles are similar and write a similarity statement. *Explain* your reasoning.

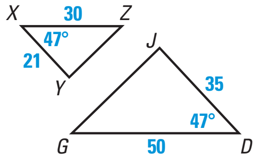
10.



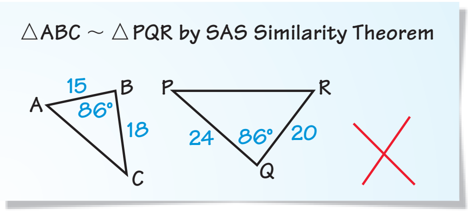
11.



12.



13. ERROR ANALYSIS *Describe* and correct the student's error in writing the similarity statement.

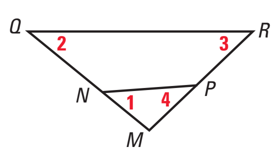


***Sample answer:* The triangle correspondence is not listed in the correct order; $\triangle ABC \sim \triangle RQP$.**

14. ★ **MULTIPLE CHOICE** In the diagram, $\frac{MN}{MR} = \frac{MP}{MQ}$. Which of the statements must be true? ***D***

Ⓐ $\angle 1 \cong \angle 2$ Ⓑ $\overline{QR} \parallel \overline{NP}$

Ⓒ $\angle 1 \cong \angle 4$ Ⓓ $\triangle MNP \sim \triangle MRQ$



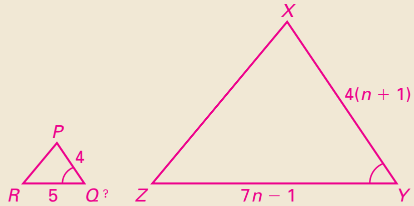
○ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

10. ***$\triangle GHJ \sim \triangle FHK$; $\frac{FH}{GH} = \frac{HK}{HJ} = \frac{KF}{JG} = \frac{4}{3}$ thus the triangles are similar by the SSS Similarity Theorem.***

11. ***$\triangle ABC \sim \triangle DEC$; $\angle ACB \cong \angle DCE$ by the Vertical Angles Congruence Theorem and $\frac{AC}{DC} = \frac{BC}{EC} = \frac{3}{2}$. The triangles are similar using the SAS Similarity Theorem.***

12. ***$\triangle XYZ \sim \triangle DJG$; $\angle D \cong \angle X$ and $\frac{DG}{XZ} = \frac{DJ}{XY} = \frac{5}{3}$. The triangles are similar by the SAS Similarity Theorem.*** B

9.



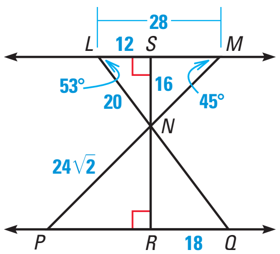
DRAWING TRIANGLES **Sketch the triangles using the given description. *Explain* whether the two triangles can be similar.** 15–17. See margin for art.

15. In $\triangle XYZ$, $m\angle X = 66°$ and $m\angle Y = 34°$. In $\triangle LMN$, $m\angle M = 34°$ and $m\angle N = 80°$. They are similar by the AA Similarity Postulate.

16. In $\triangle RST$, $RS = 20$, $ST = 32$, and $m\angle S = 16°$. In $\triangle FGH$, $GH = 30$, $HF = 48$, and $m\angle H = 24°$. They are not similar since the larger side in $\triangle RST$ would not be opposite the largest angle.

17. The side lengths of $\triangle ABC$ are 24, $8x$, and 54, and the side lengths of $\triangle DEF$ are 15, 25, and $7x$. They are not similar since the ratio of corresponding sides is not constant for any arrangement of side lengths.

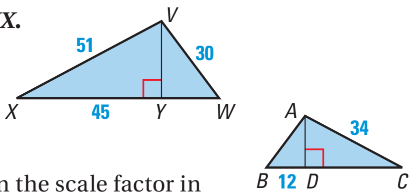
FINDING MEASURES **In Exercises 18–23, use the diagram to copy and complete the statements.**



18. $m\angle NQP = \underline{\ ?\ }$ 53°
19. $m\angle QPN = \underline{\ ?\ }$ 45°
20. $m\angle PNQ = \underline{\ ?\ }$ 82°
21. $RN = \underline{\ ?\ }$ 24
22. $PQ = \underline{\ ?\ }$ 42
23. $NM = \underline{\ ?\ }$ $16\sqrt{2}$

24. **SIMILAR TRIANGLES** In the diagram at the right, name the three pairs of triangles that are similar. $\triangle NSM$ and $\triangle NRP$, $\triangle NSL$ and $\triangle NRQ$, $\triangle NLM$ and $\triangle NQP$

C **CHALLENGE** **In the figure at the right, $\triangle ABC \sim \triangle VWX$.**



25. Find the scale factor of $\triangle VWX$ to $\triangle ABC$. $\frac{3}{2}$

26. Find the ratio of the area of $\triangle VWX$ to the area of $\triangle ABC$. $\frac{9}{4}$

27. Make a conjecture about the relationship between the scale factor in Exercise 25 and the ratio in Exercise 26. *Justify* your conjecture.

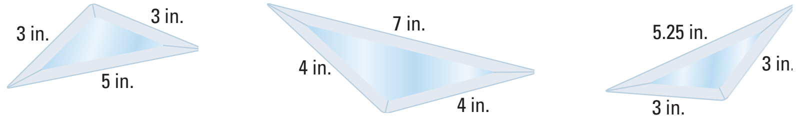
27. In similar triangles the ratio of the areas is the square of the scale factor. *Sample answer:* Let the base and height of $\triangle VWX$ measure $3a$ and $3b$ and the base and height of $\triangle ABC$ measure $2a$ and $2b$. The ratio of their areas is $\dfrac{\frac{3a(3b)}{2}}{\frac{2a(2b)}{2}} = \frac{9}{4}$.

PROBLEM SOLVING

A 28. **RACECAR NET** Which postulate or theorem could you use to show that the three triangles that make up the racecar window net are similar? *Explain*.

AA Similarity Postulate; in $\triangle AGB$ $\angle A$ and $\angle AGB$ are congruent to $\angle A$ and $\angle AFC$ in $\triangle AFC$. In $\triangle AFC$ $\angle A$ and $\angle AFC$ are congruent to $\angle A$ and $\angle AED$ in $\triangle AED$.

29. **STAINED GLASS** Certain sections of stained glass are sold in triangular *beveled* pieces. Which of the three beveled pieces, if any, are similar?



29. The triangle whose sides measure 4 inches, 4 inches, and 7 inches is similar to the triangle whose sides measure 3 inches, 3 inches, and 5.25 inches.

EXAMPLE 1 for Ex. 29

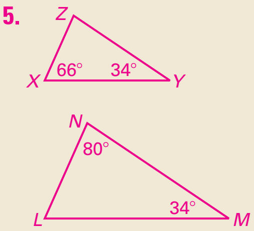
Teaching Strategy

Exercises 5, 6, 10–12, 15–17 Ask students to summarize the exercises by writing a paragraph explanation of when two triangles are similar or not similar.

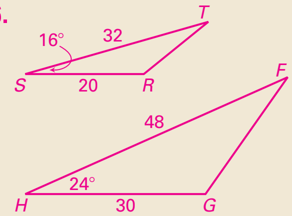
Avoiding Common Errors

Exercises 11, 18–23 Students may have difficulty writing the correct ratio of corresponding lengths for these pairs of triangles. Encourage them to redraw the triangles separately and re-label them.

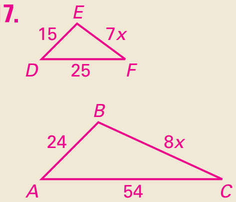
15.



16.



17.



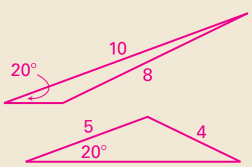
Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Exercise 33**. This activity is also part of **Power Presentations**.

Mathematical Reasoning

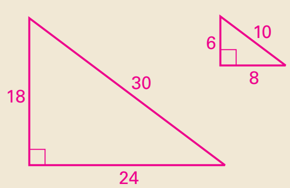
Exercise 34 In part (a), the Pythagorean Theorem will help students see that the ratio of the third pair of corresponding side lengths is the same as the other ratios. Therefore, the triangles are similar by the SSS Similarity Theorem.

32.



$\triangle XYW$ is not similar to $\triangle XZW$.

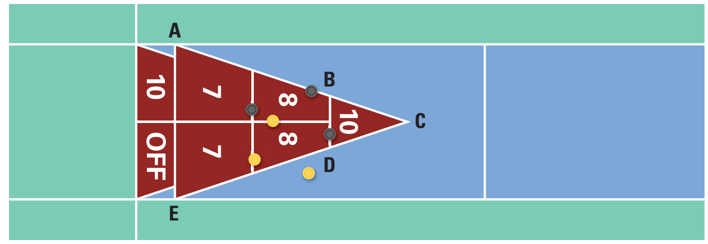
34a.



36. Yes. *Sample answer:* All pairs of similar triangles have angle pairs whose measures are in proportion (with constant of proportionality 1).

37. *Sample answer:* Locate G on $\overline{AB}$ so that $GB = DE$. Draw $\overline{GH}$ so that $\overline{GH} \parallel \overline{AC}$. This makes $\triangle ABC \sim \triangle GBH$ by the AA Similarity Postulate. From this similarity you have $\frac{AB}{GB} = \frac{AC}{GH}$. This along with what's given you get $\frac{GH}{DF} = \frac{GB}{DE}$ which implies that $\overline{GH} \cong \overline{DF}$, making $\triangle GBH \cong \triangle DEF$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude $\triangle ABC \sim \triangle DEF$.

SHUFFLEBOARD In the portion of the shuffleboard court shown, $\frac{BC}{AC} = \frac{BD}{AE}$.



30. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SSS Similarity Theorem?

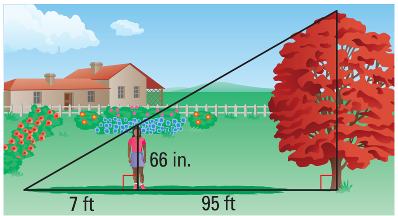
30. $\frac{CD}{CE}$ is the same scale factor as the other ratio.

31. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SAS Similarity Theorem? $\angle CBD \cong \angle CAE$

32. ★ **OPEN-ENDED MATH** Use a diagram to show why there is no Side-Side-Angle Similarity Postulate. **See margin.**

EXAMPLE 4 B
for Ex. 33

33. MULTI-STEP PROBLEM Ruby is standing in her back yard and she decides to estimate the height of a tree. She stands so that the tip of her shadow coincides with the tip of the tree's shadow, as shown. Ruby is 66 inches tall. The distance from the tree to Ruby is 95 feet and the distance between the tip of the shadows and Ruby is 7 feet.



a. What postulate or theorem can you use to show that the triangles in the diagram are similar? **AA Similarity Postulate**

b. About how tall is the tree, to the nearest foot? **80 ft**

c. What If? Curtis is 75 inches tall. At a different time of day, he stands so that the tip of his shadow and the tip of the tree's shadow coincide, as described above. His shadow is 6 feet long. How far is Curtis from the tree? **70.8 ft**

Animated Geometry at my.hrw.com

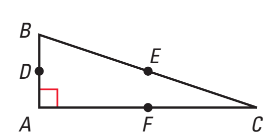
34. ★ **EXTENDED RESPONSE** Suppose you are given two right triangles with one pair of corresponding legs and the pair of corresponding hypotenuses having the same length ratios.

a. The lengths of the given pair of corresponding legs are 6 and 18, and the lengths of the hypotenuses are 10 and 30. Use the Pythagorean Theorem to solve for the lengths of the other pair of corresponding legs. Draw a diagram. **8, 24; see margin for art.**

b. Write the ratio of the lengths of the second pair of corresponding legs. $\frac{1}{3}$

c. Are these triangles similar? Does this suggest a Hypotenuse-Leg Similarity Theorem for right triangles? **yes; yes**

35. PROOF Given that $\triangle ABC$ is a right triangle and D, E, and F are midpoints, prove that $m\angle DEF = 90°$.



35. *Sample answer:* Given that D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively the Midsegment Theorem guarantees that $\overline{AC} \parallel \overline{DE}$. By the Corresponding Angles Postulate $\angle A \cong \angle BDE$ and so $\angle BDE$ is a right angle. Reasoning similarly $\overline{AB} \parallel \overline{EF}$. By the Alternate Interior Angles Congruence Theorem $\angle BDE \cong \angle DEF$. This makes $\angle DEF$ a right angle that measures 90°.

36. ★ **WRITING** Can two triangles have all pairs of corresponding angles in proportion? *Explain.* **See margin.**

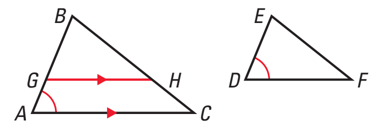
◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

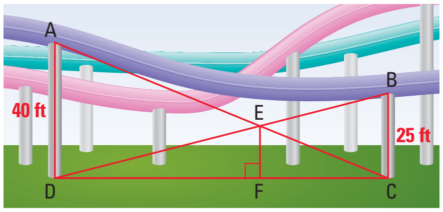
37. **PROVING THEOREM 6.3** Write a paragraph proof of the SAS Similarity Theorem. **See margin.**

GIVEN ▶ $\angle A \cong \angle D$, $\frac{AB}{DE} = \frac{AC}{DF}$

PROVE ▶ $\triangle ABC \sim \triangle DEF$



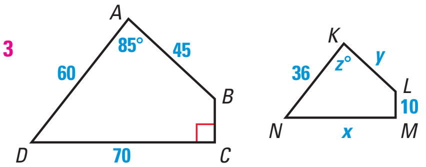
[C] 38. **CHALLENGE** A portion of a water slide in an amusement park is shown. Find the length of $\overline{EF}$. (*Note:* The posts form right angles with the ground.) **about 15.4 ft**



QUIZ

In the diagram, *ABCD* ~ *KLMN*.

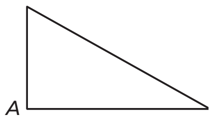
1. Find the scale factor of *ABCD* to *KLMN*. **5 : 3**
2. Find the values of *x*, *y*, and *z*. **42, 27, 85**
3. Find the perimeter of each polygon. **$191\frac{2}{3}$, 115**



Copy the figure. Draw the indicated similarity transformation of the figure with center *A*.

4. dilation

5. dilation then reflection

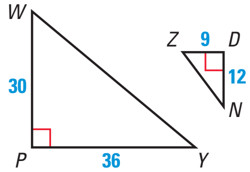


6. dilation then rotation

4–6. Check students' drawings.

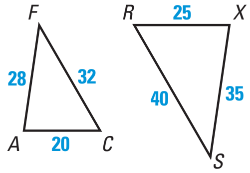
Determine whether the triangles are similar. If they are similar, write a similarity statement.

7.



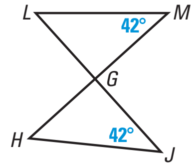
not similar

8.



similar; $\triangle ACF \sim \triangle XRS$

9.



similar; $\triangle MGL \sim \triangle JGH$

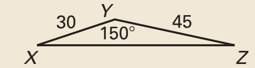
5 ASSESS AND RETEACH

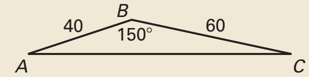
Daily Homework Quiz

Also available online

1. Verify that $\triangle ABC \sim \triangle DEF$ for the given information.
$\triangle ABC$: $AC = 6$, $AB = 9$, $BC = 12$;
$\triangle DEF$: $DF = 2$, $DE = 3$, $EF = 4$
$\frac{AC}{DF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{3}{1}$. The ratios are equal, so $\triangle ABC \sim \triangle DEF$ by the SSS Similarity Theorem.

2. Show that the triangles are similar and write a similarity statement. Explain your reasoning.





$\frac{XY}{AB} = \frac{YZ}{BC} = \frac{3}{4}$ and $\angle Y \cong \angle B$. So $\triangle XYZ \sim \triangle ABC$ by the SAS Similarity Theorem.

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

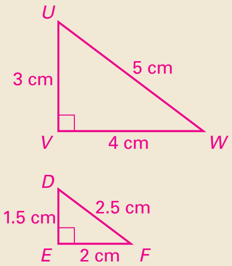
Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

1. *Sample:*



3. Translate *LMN* so that *L* coincides with *Q*. Dilate *LMN* so that *M* coincides with *R*. Then *N* will coincide with *S*.

4. Yes. Translate *LMN* so that *L* coincides with *S*. Reflect *LMN* so that $\overline{LM}$ is colinear with $\overline{SR}$. Dilate *LMN* so that *M* coincides with *R*. Then *N* will coincide with *Q*.

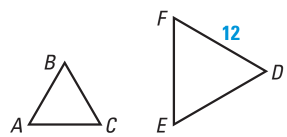
5. No. Translate *LMN* so that *L* coincides with *Q*. Reflect *LMN* so that $\overline{LM}$ is colinear with $\overline{QS}$. Dilate *LMN* so that *M* coincides with *S*. Then *N* will not coincide with *R*.

6a. Use the AA Similarity Postulate since $\angle D$ and $\angle B$ are right angles and $\angle ACB \cong \angle ECD$ by the Vertical Angles Congruence Theorem.

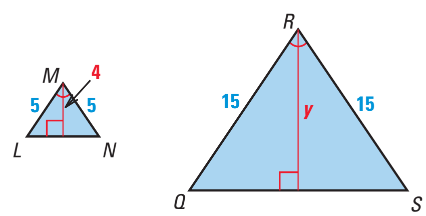
MIXED REVIEW *of Problem Solving*

1. **OPEN-ENDED** $\triangle UVW$ is a right triangle with side lengths of 3 cm, 4 cm, and 5 cm. Draw and label $\triangle UVW$. Then draw a triangle similar to $\triangle UVW$ and label its side lengths. What scale factor did you use? **See margin for art.** ***Sample answer:*** $\frac{1}{2}$

2. **GRIDDED ANSWER** In the diagram, $\triangle ABC \sim \triangle DEF$. The scale factor of $\triangle ABC$ to $\triangle DEF$ is 3 : 5. Find *AC*. **7.2**



For Exercises 3–5, use the triangles below.



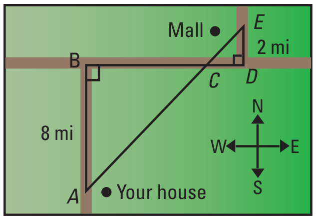
3. Which postulate or theorem could you use to prove that the diagram shows two similar triangles? **AA Similarity Postulate or SAS Similarity Theorem**

4. Which transformation(s) could you use to show that *LMN* is similar to *SRQ*? **dilation and translation**

5. Which transformation(s) could you use to show that *LMN* is similar to *QSR*? **dilation, reflection, and translation**

6. **OPEN-ENDED** The diagram shows the front of a house. What information would you need in order to show that $\triangle WXY \sim \triangle VXZ$ using the SAS Similarity Theorem? $\frac{XW}{XV} = \frac{XY}{XZ}$

7. **EXTENDED RESPONSE** You leave your house to go to the mall. You drive due north 8 miles, due east 7.5 miles, and due north again 2 miles.



a. *Explain* how to prove that $\triangle ABC \sim \triangle EDC$. **See margin.**

b. Find *CD*. **1.5 mi**

c. Find *AE*, the distance between your house and the mall. **12.5 mi**

6.5 Use Proportionality Theorems

Before You used proportions with similar triangles.
Now You will use proportions with a triangle or parallel lines.
Why? So you can use perspective drawings, as in Ex. 28.

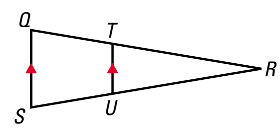
Key Vocabulary
- corresponding angles
- ratio
- proportion

The Midsegment Theorem is a special case of the Triangle Proportionality Theorem and its converse.

THEOREMS *For Your Notebook*

THEOREM 6.4 Triangle Proportionality Theorem

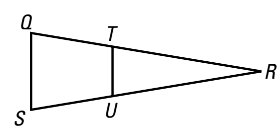
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

THEOREM 6.5 Converse of the Triangle Proportionality Theorem

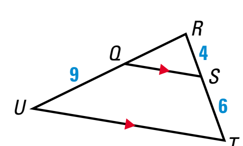
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

EXAMPLE 1 Find the length of a segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?



Solution

$\frac{RQ}{QU} = \frac{RS}{ST}$ Triangle Proportionality Theorem

$\frac{RQ}{9} = \frac{4}{6}$ Substitute.

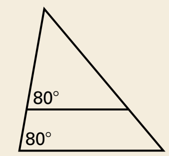
$RQ = 6$ Multiply each side by 9 and simplify.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. Give the postulate or theorem that justifies why the triangles are similar.



AA Similarity Postulate

2. Solve $\frac{16}{32} = \frac{12 - x}{x}$. **8**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

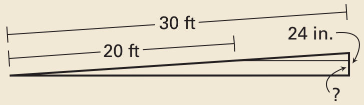
Essential Question

Big Idea 1

What proportion can you write if a line is parallel to one side of a triangle? **Tell students they will learn how to answer this question by using a generalization of the Midsegment Theorem.**

Motivating the Lesson

Show students this diagram of a wheelchair ramp.

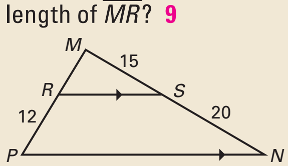


Ask them what vertical distance corresponds with going up 20 feet on the ramp. Discuss with them that $\frac{2}{3}$ of the ramp distance corresponds with $\frac{2}{3}$ of the vertical height. Tell students that in this lesson they will investigate other properties related to a line parallel to one side of a triangle.

3 TEACH

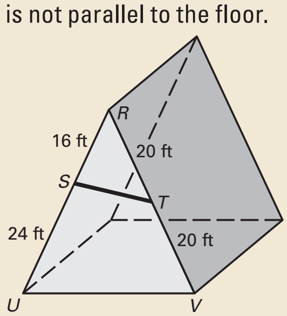
Extra Example 1

In the diagram, $\overline{RS} \parallel \overline{PN}$, $MS = 15$, $SN = 20$, and $RP = 12$. What is the length of $\overline{MR}$? **9**



Extra Example 2

A brace is added to a tree house as shown. Explain why the brace is not parallel to the floor.



$\frac{RS}{SU} \neq \frac{RT}{TV}$ so $\overline{ST}$ is not parallel to $\overline{UV}$ and the brace is not parallel to the floor.

REVIEW CONTRAPOSITIVES
When an if-then statement is true, its *contrapositive* is also true

REASONING Theorems 6.4 and 6.5 also imply the following:

Contrapositive of Theorem 6.4

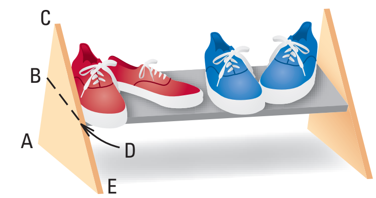
If $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \not\parallel \overline{QS}$.

Contrapositive of Theorem 6.5

If $\overline{TU} \not\parallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$.

EXAMPLE 2 Solve a real-world problem

SHOERACK On the shoerack shown, $AB = 33$ cm, $BC = 27$ cm, $CD = 44$ cm, and $DE = 25$ cm. *Explain* why the gray shelf is not parallel to the floor.



Solution

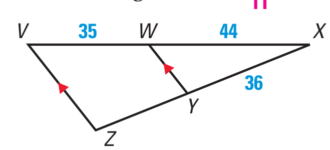
Find and simplify the ratios of lengths determined by the shoerack.

$$\frac{CD}{DE} = \frac{44}{25} \qquad \frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$$

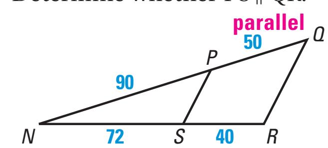
▸ Because $\frac{44}{25} \neq \frac{9}{11}$, $\overline{BD}$ is not parallel to $\overline{AE}$. So, the shelf is not parallel to the floor.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the length of $\overline{YZ}$. $\frac{315}{11}$



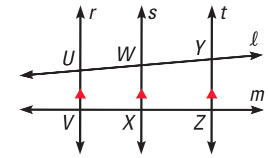
2. Determine whether $\overline{PS} \parallel \overline{QR}$. parallel



THEOREMS *For Your Notebook*

THEOREM 6.6

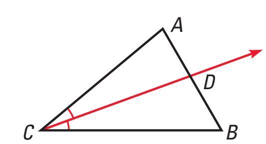
If three parallel lines intersect two transversals, then they divide the transversals proportionally.



$$\frac{UW}{WY} = \frac{VX}{XZ}$$

THEOREM 6.7

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



$$\frac{AD}{DB} = \frac{CA}{CB}$$

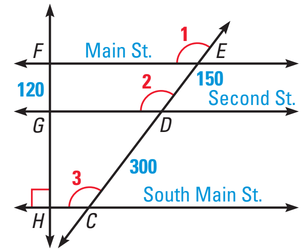
Differentiated Instruction

Auditory Learners Have students complete **Guided Practice Exercises 1 and 2** with a partner. Instruct them to talk about the problem as they work through it. They should work together to describe the steps they are taking to solve the problem as if they were describing a proof.

See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 3 Use Theorem 6.6

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



Solution

Corresponding angles are congruent, so $\overleftrightarrow{FE}$, $\overleftrightarrow{GD}$, and $\overleftrightarrow{HC}$ are parallel. Use Theorem 6.6.

$\frac{HG}{GF} = \frac{CD}{DE}$ **Parallel lines divide transversals proportionally.**

$\frac{HG + GF}{GF} = \frac{CD + DE}{DE}$ **Property of proportions**

$\frac{HF}{120} = \frac{300 + 150}{150}$ **Substitute.**

$\frac{HF}{120} = \frac{450}{150}$ **Simplify.**

$HF = 360$ **Multiply each side by 120 and simplify.**

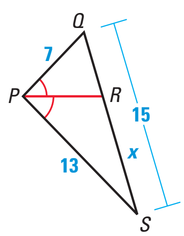
▶ The distance between Main Street and South Main Street is 360 yards.

ANOTHER WAY

For alternative methods for solving the problem in Example 3, see the **Problem Solving Workshop**.

EXAMPLE 4 Use Theorem 6.7

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.



Solution

Because $\overrightarrow{PR}$ is an angle bisector of $\angle QPS$, you can apply Theorem 6.7. Let $RS = x$. Then $RQ = 15 - x$.

$\frac{RQ}{RS} = \frac{PQ}{PS}$ **Angle bisector divides opposite side proportionally.**

$\frac{15 - x}{x} = \frac{7}{13}$ **Substitute.**

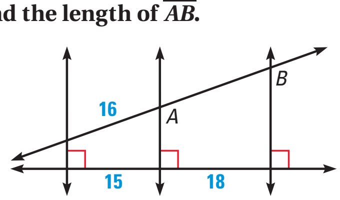
$7x = 195 - 13x$ **Cross Products Property**

$x = 9.75$ **Solve for *x*.**

✓ GUIDED PRACTICE for Examples 3 and 4

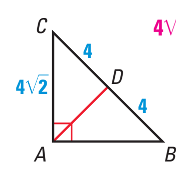
Find the length of $\overline{AB}$.

3.



19.2

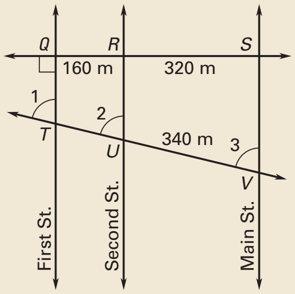
4.



$4\sqrt{2}$

Extra Example 3

Using the information in the diagram, find the distance between First Street and Main Street.



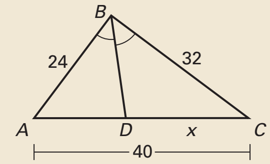
510 m

Key Question to Ask for Example 3

- Why is it important that $\angle 1 \cong \angle 2 \cong \angle 3$? **The streets need to be parallel to apply Theorem 6.6.**

Extra Example 4

In the diagram, $\angle ABD \cong \angle CBD$. Use the given side lengths to find the length of $\overline{DC}$.



$22\frac{6}{7}$

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What proportion can you write if a line is parallel to one side of a triangle?

- **A line parallel to one side of a triangle intersecting the other sides divides those sides proportionally, and vice versa.**
- **Three parallel lines intersecting two transversals divide the transversals proportionally.**
- **An angle bisector of a triangle divides the opposite sides into segments whose lengths are proportional to the lengths of the other sides.**

In $\triangle ABC$, with D on $\overline{AB}$ and E on $\overline{AC}$, if $\overline{DE} \parallel \overline{BC}$ then $\frac{AD}{DB} = \frac{AE}{EC}$.

Differentiated Instruction

Below Level To help students remember how to write the proportion for Theorem 6.7, ask them to label $\overline{CA}$ as Side 1, $\overline{AD}$ as Part 1, $\overline{CB}$ as Side 2, and $\overline{DB}$ as Part 2. They should notice that Side 1 is adjacent to Part 1, and Side 2 is adjacent to Part 2. Then the correct proportion is $\frac{\text{Side 1}}{\text{Side 2}} = \frac{\text{Part 1}}{\text{Part 2}}$.

See also the *Differentiated Instruction Resources* for more strategies.

6.5 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 9, and 21
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 8, 13, 25, and 28

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–7, 13, 16, 22
Day 2:
Exs. 8–12, 14, 15, 21, 23–26

Average:
Day 1:
Exs. 1–7, 13, 16, 22
Day 2:
Exs. 8–12, 14, 15, 17–19, 21, 23–28

Advanced:
Day 1:
Exs. 1–7, 13, 16, 22
Day 2:
Exs. 8, 10, 11, 14, 15, 17–21*, 23–29*

Block:
Exs. 1–19, 21–28

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 8, 10, 22
Average: 4, 7, 8, 10, 23
Advanced: 7, 11, 16, 24, 25

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

SKILL PRACTICE

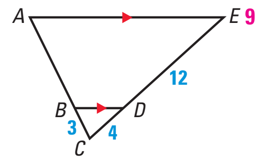
A

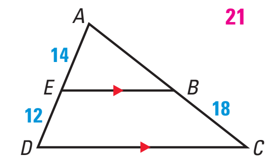
1. **VOCABULARY** State the Triangle Proportionality Theorem. Draw a diagram. **See margin.**

2. ★ **WRITING** *Compare* the Midsegment Theorem and the Triangle Proportionality Theorem. How are they related? **See margin.**

EXAMPLE 1 for Exs. 3–4

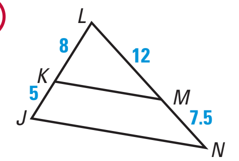
FINDING THE LENGTH OF A SEGMENT Find the length of $\overline{AB}$.

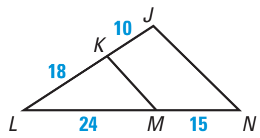
3. 

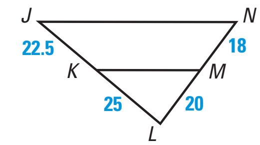
4. 

EXAMPLE 2 for Exs. 5–7

REASONING Use the given information to determine whether $\overline{KM} \parallel \overline{JN}$. *Explain* your reasoning. **5–7. See margin.**

5. 

6. 

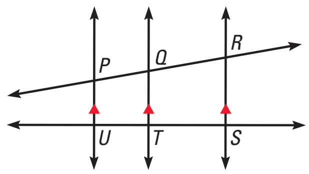
7. 

EXAMPLE 3 for Ex. 8

8. ★ **MULTIPLE CHOICE** For the figure at the right, which statement is *not* necessarily true? **C**

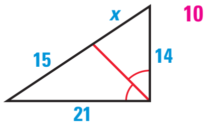
Ⓐ $\frac{PQ}{QR} = \frac{UT}{TS}$ Ⓑ $\frac{TS}{UT} = \frac{QR}{PQ}$

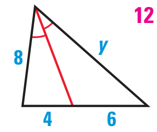
Ⓒ $\frac{QR}{RS} = \frac{TS}{RS}$ Ⓓ $\frac{PQ}{PR} = \frac{UT}{US}$

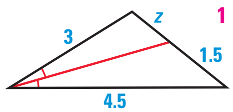


EXAMPLE 4 for Exs. 9–12

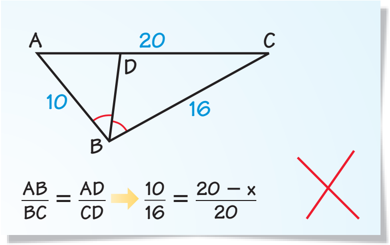
ALGEBRA Find the value of the variable.

9. 

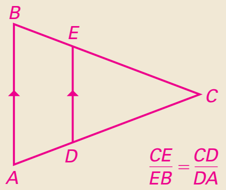
10. 

11. 

12. **ERROR ANALYSIS** A student begins to solve for the length of $\overline{AD}$ as shown. *Describe* and correct the student's error.
The length of $\overline{CD}$ is not 20; $\frac{10}{16} = \frac{20-x}{x}$.



1. If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

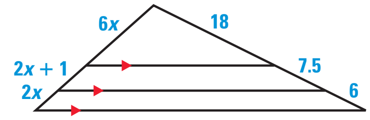


2. In the Midsegment Theorem, the segment connecting the midpoints of two sides of a triangle is parallel to the third side, which is a special case of the Converse of the Triangle Proportionality Theorem.

B 13. ★ **MULTIPLE CHOICE** Find the value of x. **C**

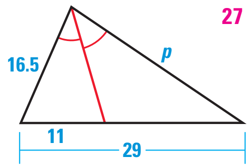
Ⓐ $\frac{1}{2}$ Ⓑ 1

Ⓒ 2 Ⓓ 3

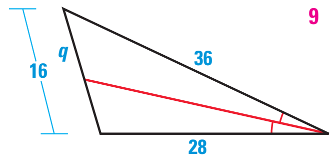


ALGEBRA **Find the value of the variable.**

14. **27**

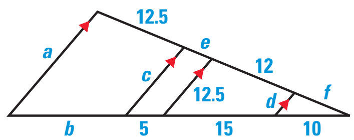


15. **9**



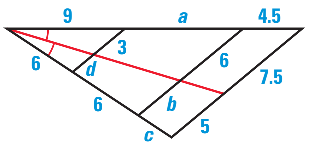
FINDING SEGMENT LENGTHS **Use the diagram to find the value of each variable.**

16.



$a = 22.8125$, $b = 15.625$, $c = 15$, $d = 5$, $e = 4$, $f = 8$

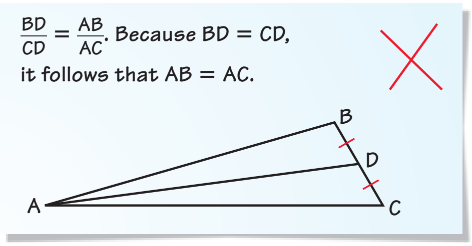
17.



$a = 9$, $b = 4$, $c = 3$, $d = 2$

18. **ERROR ANALYSIS** A student claims that $AB = AC$ using the method shown. *Describe* and correct the student's error.

The student cannot make the claim because it is not given that $\overline{AD}$ bisects $\angle A$.



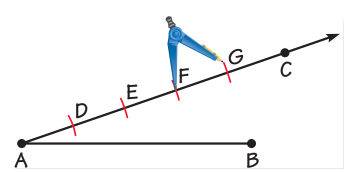
19c. See margin for art; parallel lines divide transversals proportionally. Since $\frac{AD}{DE} = \frac{DE}{EF} = \frac{EF}{FG} = 1$ implies $\frac{AJ}{JK} = \frac{JK}{KL} = \frac{KL}{LB} = 1$ which means $AJ = JK = KL = LB$.

19. **CONSTRUCTION** Follow the instructions for constructing a line segment that is divided into four equal parts.

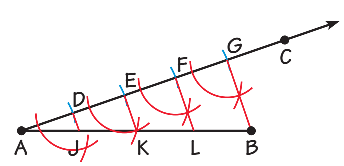
a. Draw a line segment that is about 3 inches long, and label its endpoints A and B. Choose any point C not on $\overline{AB}$. Draw $\overrightarrow{AC}$.

a, b. See figure in part (c).

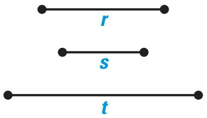
b. Using any length, place the compass point at A and make an arc intersecting $\overrightarrow{AC}$ at D. Using the same compass setting, make additional arcs on $\overrightarrow{AC}$. Label the points E, F, and G so that $AD = DE = EF = FG$.



c. Draw $\overline{GB}$. Construct a line parallel to $\overline{GB}$ through D. Continue constructing parallel lines and label the points as shown. *Explain* why $AJ = JK = KL = LB$.



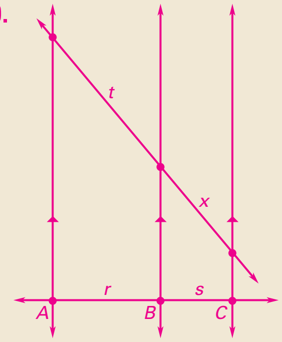
C **20.** **CHALLENGE** Given segments with lengths r, s, and t, construct a segment of length x, such that $\frac{r}{s} = \frac{t}{x}$. **See margin.**



Vocabulary

Exercise 19 Students may need to review how to identify corresponding angles and that if corresponding angles are congruent, then the lines are parallel.

20.

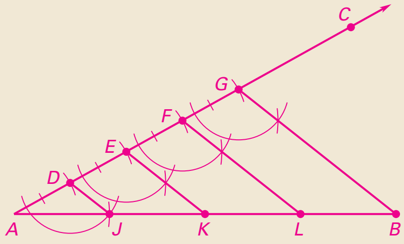


5. Parallel; $\frac{8}{5} = \frac{12}{7.5}$, so the Converse of the Triangle Proportionality Theorem applies.

6. not parallel; $\frac{24}{15} \neq \frac{18}{10}$

7. Parallel; $\frac{20}{18} = \frac{25}{22.5}$, so the Converse of the Triangle Proportionality Theorem applies.

19c.



Teaching Strategy

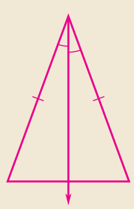
Exercises 23, 27 Point out that an auxiliary line added to the diagram allows you to identify and use the proportionality theorems for triangles and parallel lines.

Internet Reference

Exercise 28 More information about perspective drawing can be found at mathforum.org/sum95/math_and/perspective/perspect.html

23. Draw $\overleftrightarrow{AD}$. (Through any two points, there is exactly one line.) Let G be the point of intersection of $\overleftrightarrow{AD}$ and $\overleftrightarrow{BE}$. Since $k_1 \parallel k_2$ and $k_2 \parallel k_3$, by the Triangle Proportionality Theorem $\frac{CB}{BA} = \frac{DG}{GA}$ and $\frac{DG}{GA} = \frac{DE}{EF}$. Using the Transitive Property of Equality, $\frac{CB}{BA} = \frac{DE}{EF}$.

25.



26. *Sample answer:* Begin by showing $\frac{RT + TQ}{TQ} = \frac{RU + US}{US}$ and simplifying this to $\frac{RQ}{TQ} = \frac{RS}{US}$. Use the proportions to solve for $\frac{TQ}{US}$ and use the Transitive Property of Equality. Show $\triangle RTU \sim \triangle RQS$ using the SAS Similarity Theorem and show $\angle RTU \cong \angle RQS$ by definition of similar triangles. Then use the Corresponding Angles Converse to show $\overline{QS} \parallel \overline{TU}$.

22. Since $\overline{QS} \parallel \overline{TU}$ $\angle S \cong \angle TUR$ and $\angle Q \cong \angle UTR$ using the Corresponding Angles Postulate. $\triangle SRQ \sim \triangle URT$ using the AA Similarity Postulate. $\frac{QR}{TR} = \frac{SR}{UR}$ using the definition of similarity. $QR = QT + TR$ and $SR = SU + UR$ by the Segment Addition Postulate. Substituting you get $\frac{QT + TR}{TR} = \frac{SU + UR}{UR}$ which simplifies to $\frac{QT}{TR} = \frac{SU}{UR}$.

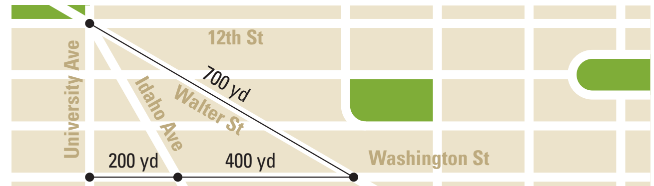
24a. Lot A = 50.9 yd, Lot B = 58.4 yd, Lot C = 64.7 yd

25. See margin for art; in an isosceles triangle, the legs are congruent, so the ratio of their lengths is 1 : 1. But this ratio is equal to the ratio of the lengths of the segments created by the ray, so it is also 1 : 1.

PROBLEM SOLVING

A

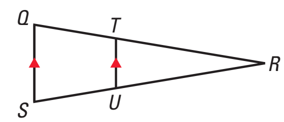
21. **CITY MAP** On the map below, Idaho Avenue bisects the angle between University Avenue and Walter Street. To the nearest yard, what is the distance along University Avenue from 12th Street to Washington Street? **350 yd**



22. **PROVING THEOREM 6.4** Prove the Triangle Proportionality Theorem.

GIVEN ▶ $\overline{QS} \parallel \overline{TU}$

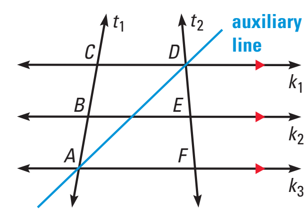
PROVE ▶ $\frac{QT}{TR} = \frac{SU}{UR}$



23. **PROVING THEOREM 6.6** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 6.6. **See margin.**

GIVEN ▶ $k_1 \parallel k_2$, $k_2 \parallel k_3$

PROVE ▶ $\frac{CB}{BA} = \frac{DE}{EF}$

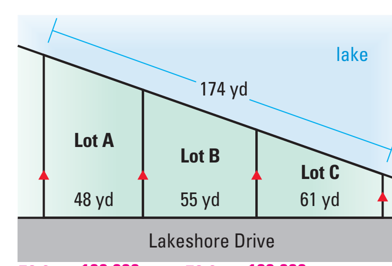


24. **MULTI-STEP PROBLEM** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.

a. Find the lake frontage (to the nearest tenth of a yard) for each lot shown.

b. In general, the more lake frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price? **Lot C**

c. Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is \$100,000, what are the prices of the other lots? *Explain* your reasoning.



About \$114,735; about \$127,112. *Sample answer:* Solve $\frac{50.9}{58.4} = \frac{100{,}000}{x}$ and $\frac{50.9}{64.7} = \frac{100{,}000}{x}$.

B

25. ★ **SHORT RESPONSE** Sketch an isosceles triangle. Draw a ray that bisects the angle opposite the base. This ray divides the base into two segments. By Theorem 6.7, the ratio of the legs is proportional to the ratio of these two segments. *Explain* why this ratio is 1 : 1 for an isosceles triangle.

26. **PLAN FOR PROOF** Use the diagram given for the proof of Theorem 6.4 in Exercise 22 to write a plan for proving Theorem 6.5, the Triangle Proportionality Converse. **See margin.**

○ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**

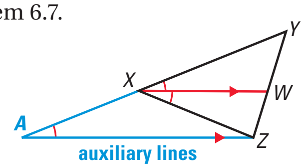
28b. *Sample answer:* The line connecting the top left to the bottom left of Car 1 is parallel to the line connecting the top left to the bottom left of Car 2; the triangle with vertices consisting of the vanishing point, the top left of Car 1, and the bottom left of Car 1 is similar to the triangle with vertices consisting of the vanishing point, the top left of Car 2, and the bottom left of Car 2.

27. Since $\overline{XW} \parallel \overline{AZ}$, $\angle XZA \cong \angle WXZ$ using the Alternate Interior Angles Congruence Theorem. This makes $\triangle AXZ$ isosceles because it is shown that $\angle A \cong \angle WXZ$ and by the Converse of the Base Angles Theorem, $AX = XZ$. Since $\overline{XW} \parallel \overline{AZ}$ using the Triangle Proportionality Theorem you get $\frac{YW}{WZ} = \frac{XY}{AX}$. Substituting you get $\frac{YW}{WZ} = \frac{XY}{XZ}$.

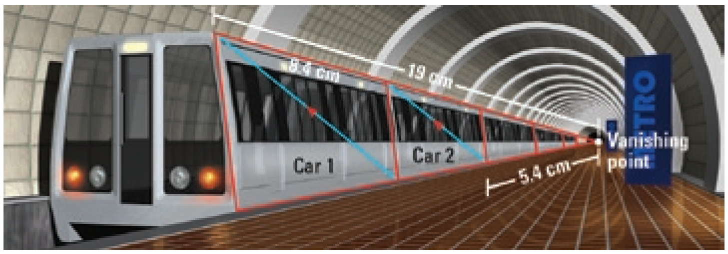
27. PROVING THEOREM 6.7 Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 6.7.

GIVEN ▶ $\angle YXW \cong \angle WXZ$

PROVE ▶ $\frac{YW}{WZ} = \frac{XY}{XZ}$



28. ★ **EXTENDED RESPONSE** In *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel.

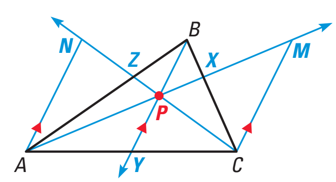


a. Use the dimensions given and the red parallel lines to find the length of the bottom edge of the drawing of Car 2. **about 4.3 cm**

b. What other set of parallel lines exist in the figure? *Explain* how these can be used to form a set of similar triangles. **See margin.**

c. Find the length of the top edge of the drawing of Car 2. **about 4.7 cm**

C **29. CHALLENGE** Prove *Ceva's Theorem:* If P is any point inside $\triangle ABC$, then $\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$. (*Hint:* Draw lines parallel to $\overline{BY}$ through A and C. Apply Theorem 6.4 to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.) **See margin.**



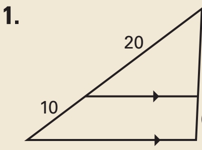
5 ASSESS AND RETEACH

Daily Homework Quiz

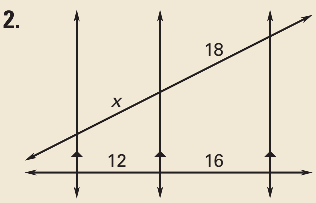
Also available online

Find the value of the variable.

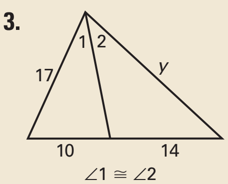
1. **12**



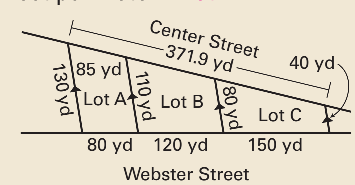
2. **13.5**



3. **23.8**



4. This diagram represents a tract of land being developed for homes. Which lot has the greatest perimeter? **Lot B**



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

29. See Additional Answers.

PROBLEM SOLVING WORKSHOP LESSON 6.5

Using ALTERNATIVE METHODS

Alternative Strategy

Example 3 in this lesson can be solved by looking for and applying ratios in the diagram (Method 1) or by using a graphic organizer to set up a proportion (Method 2).

Teaching Strategy

Point out how the graphic organizer is set up in Method 2. The total distances along $\overleftrightarrow{CE}$ and $\overleftrightarrow{HF}$ are compared to partial (shorter) distances along those streets.

Study Strategy

For Exercise 1(b), point out that if there is a parallel alley one-fourth the way from $\overline{BE}$ to $\overline{CD}$, then the alley is one-fourth the distance from E to D.

Mathematical Reasoning

In Exercise 2, parallel lines are used to prove pairs of triangles similar by the AA Similarity Postulate. Then the corresponding side lengths for the pair of triangles can be used to write a proportion.

Avoiding Common Errors

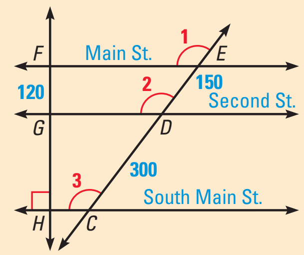
For Exercise 5, a common error is to conclude from the diagram that $\frac{2}{5}$ and $\frac{1.5}{x}$ are equal ratios. You can use the sides of similar triangles to show that the proportion should be $\frac{2}{7} = \frac{1.5}{x}$.

Another Way to Solve Example 3

MULTIPLE REPRESENTATIONS In this lesson, you used proportionality theorems to find lengths of segments formed when transversals intersect two or more parallel lines. Now, you will learn two different ways to solve Example 3.

PROBLEM

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



METHOD 1

Applying a Ratio One alternative approach is to look for ratios in the diagram.

STEP 1 **Read** the problem. Because Main Street, Second Street, and South Main Street are all parallel, the lengths of the segments of the cross streets will be in proportion, so they have the same ratio.

STEP 2 **Apply** a ratio. Notice that on $\overleftrightarrow{CE}$, the distance CD between South Main Street and Second Street is twice the distance DE between Second Street and Main Street. So the same will be true for the distances HG and GF.

$HG = 2 \cdot GF$	Write equation.
$= 2 \cdot 120$	Substitute.
$= 240$	Simplify.

STEP 3 **Calculate** the distance. Line HF is perpendicular to both Main Street and South Main Street, so the distance between Main Street and South Main Street is this perpendicular distance, HF.

$HF = HG + GF$	Segment Addition Postulate
$= 120 + 240$	Substitute.
$= 360$	Simplify.

STEP 4 **Check** Example 3 within this lesson to verify your answer, and confirm that it is the same.

METHOD 2 **Writing a Proportion** Another alternative approach is to use a graphic organizer to set up a proportion.

STEP 1 **Make** a table to compare the distances.

	$\overleftrightarrow{CE}$	$\overleftrightarrow{HF}$
Total distance	300 + 150, or 450	x
Partial distance	150	120

STEP 2 **Write** and solve a proportion.

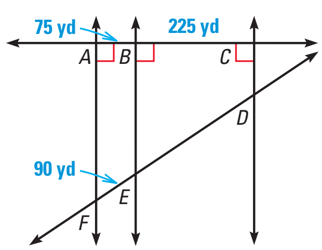
$\frac{450}{150} = \frac{x}{120}$ **Write proportion.**

$360 = x$ **Multiply each side by 120 and simplify.**

▶ The distance is 360 yards.

PRACTICE

1. MAPS Use the information on the map.



a. Find DE. **270 yd**

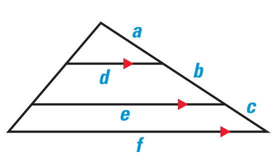
b. What If? Suppose there is an alley one fourth of the way from $\overline{BE}$ to $\overline{CD}$ and parallel to $\overline{BE}$. What is the distance from E to the alley along $\overleftrightarrow{FD}$? **67.5 yd**

2. REASONING Given the diagram below, *explain* why the three given proportions are true.

$$\frac{a}{a+b} = \frac{d}{e}$$

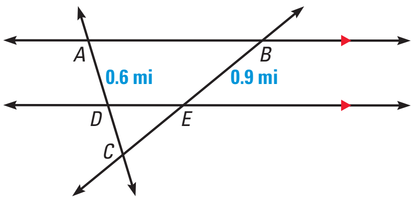
$$\frac{a}{a+b+c} = \frac{d}{f}$$

$$\frac{a+b}{a+b+c} = \frac{e}{f}$$



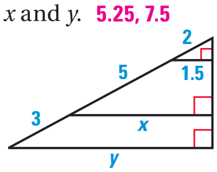
Since the three triangles are similar, the ratio of corresponding sides is the same.

3. WALKING Two people leave points A and B at the same time. They intend to meet at point C at the same time. The person who leaves point A walks at a speed of 3 miles per hour. How fast must the person who leaves point B walk? **4.5 mi/h**



4. ERROR ANALYSIS A student who attempted to solve the problem in Exercise 3 claims that you need to know the length of $\overline{AC}$ to solve the problem. *Describe* and correct the error that the student made. **You only need to know that the ratio of the two distances is 2 : 3.**

5. xy **ALGEBRA** Use the diagram to find the values of x and y. **5.25, 7.5**



1 PLAN AND PREPARE

Warm-Up Exercises

1. Simplify 3^n for $n = 0, 1, 2, 3,$ and 4. **1, 3, 9, 27, 81**

2. Evaluate $3 \cdot 4^n$ for $n = 5$. **3072**

2 FOCUS AND MOTIVATE

Essential Question

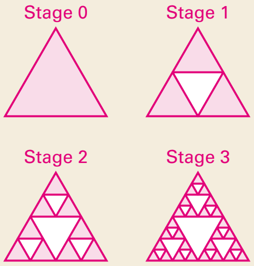
Big Idea 3

What is one way to generate a fractal? **Tell students they will learn how to answer this question by drawing a Stage 0 shape and then repeating the sequence of steps that define the fractal.**

3 TEACH

Extra Example 1

Draw a Sierpinski triangle: Start with an equilateral triangle. At each stage connect the midpoints of each side, forming a new equilateral triangle. Shade the triangle in the center with a different color.



Extension Fractals

GOAL Explore the properties of fractals.

Key Vocabulary
- fractal
- self-similarity
- iteration

HISTORY NOTE

Computers made it easier to study mathematical iteration by reducing the time needed to perform calculations. Using fractals, mathematicians have been able to create better models of coastlines, clouds, and other natural objects.

A **fractal** is an object that is *self-similar*. An object is **self-similar** if one part of the object can be enlarged to look like the whole object. In nature, fractals can be found in ferns and branches of a river. Scientists use fractals to map out clouds in order to predict rain.

A Mandelbrot fractal

Many fractals are formed by a repetition of a sequence of the steps called **iteration**. The first stage of drawing a fractal is considered Stage 0. Helge van Koch (1870–1924) described a fractal known as the *Koch snowflake*, shown in Example 1.

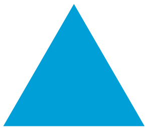
EXAMPLE 1 Draw a fractal

Use the directions below to draw a Koch snowflake.

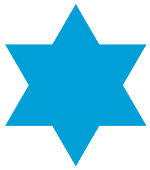
Starting with an equilateral triangle, at each stage each side is divided into thirds and a new equilateral triangle is formed using the middle third as the triangle side length.

Solution

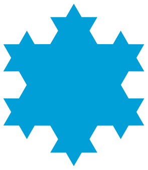
STAGE 0 **Draw** an equilateral triangle with a side length of one unit.



STAGE 1 **Replace** the middle third of each side with an equilateral triangle.



STAGE 2 **Repeat** Stage 1 with the six smaller equilateral triangles.



STAGE 3 **Repeat** Stage 1 with the eighteen smaller equilateral triangles.

MEASUREMENT Benoit Mandelbrot (b. 1924) was the first mathematician to formalize the idea of fractals when he observed methods used to measure the lengths of coastlines. Coastlines cannot be measured as straight lines because of the inlets and rocks. Mandelbrot used fractals to model coastlines.

EXAMPLE 2 Find lengths in a fractal

Make a table to study the lengths of the sides of a Koch snowflake at different stages.

Stage number	Edge length	Number of edges	Perimeter
0	1	3	3
1	$\frac{1}{3}$	$3 \cdot 4 = 12$	4
2	$\frac{1}{9}$	$12 \cdot 4 = 48$	$\frac{48}{9} = 5\frac{1}{3}$
3	$\frac{1}{27}$	$48 \cdot 4 = 192$	$\frac{192}{27} = 7\frac{1}{9}$
n	$\frac{1}{3^n}$	$3 \cdot 4^n$	$\frac{4^n}{3^{n-1}}$

Animated Geometry at my.hrw.com

PRACTICE

EXAMPLES 1 and 2 for Exs. 1–3

1. **PERIMETER** Find the ratio of the edge length of the triangle in Stage 0 of a Koch snowflake to the edge length of the triangle in Stage 1. How is the perimeter of the triangle in Stage 0 related to the perimeter of the triangle in Stage 1? *Explain.* **3 : 1. *Sample answer:* It's one unit longer; each of the three edges went from measuring one unit to four edges each measuring $\frac{1}{3}$ of a unit.**

2. **MULTI-STEP PROBLEM** Use the *Cantor set*, which is a fractal whose iteration consists of dividing a segment into thirds and erasing the middle third.
   a. Draw Stage 0 through Stage 5 of the Cantor set. Stage 0 has a length of one unit. **a, b. See margin.**
   b. Make a table showing the stage number, number of segments, segment length, and total length of the Cantor set.
   c. What is the total length of the Cantor set at Stage 10? Stage 20? Stage n?

2c. $\frac{1024}{59049}$, or about 0.01734 units; $\frac{1048576}{3486784401}$, or about 0.0003007 units; $\left(\frac{2}{3}\right)^n$

3. **EXTENDED RESPONSE** A *Sierpinski carpet* starts with a square with side length one unit. At each stage, divide the square into nine equal squares with the middle square shaded a different color.
   a. Draw Stage 0 through Stage 3 of a Sierpinski carpet. **See margin.**
   b. *Explain* why the carpet is said to be *self-similar* by comparing the upper left hand square to the whole square.
   c. Make a table to find the total area of the colored squares at Stage 3. **See margin.**

3b. *Sample answer:* The upper left square is simply a smaller version of the whole square.

Extra Example 2

Make a table of the lengths of the sides of the equilateral triangles in the Sierpinski triangle.

Stage Number	Edge Length	No. of Triangles with this Edge Length
0	1	1
1	$\frac{1}{2}$	3
2	$\frac{1}{4}$	9
3	$\frac{1}{8}$	27
n	$\left(\frac{1}{2}\right)^n$	3^n

Key Question to Ask for Example 2

- In the table, why are there $3 \cdot 4$ edges after Stage 1? **Each of 3 edges of the triangle in Stage 0 is replaced with 4 edges.**

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 2**. This activity is also part of **Power Presentations**.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What is one way to generate a fractal?

- **Draw Stage 0 for the fractal.**
- **Follow the directions and draw Stage 1.**
- **Repeat to draw more stages.**

Draw Stage 0 and repeat the sequence of steps that define the fractal.

4 PRACTICE AND APPLY

Vocabulary

Exercise 3(b) The idea of "self-similar" may be new to some students. One way to describe a self-similar figure is that it is formed by replicating itself using a scale other than 1.

2a–b, 3a, 3c. See Additional Answers.

1 PLAN AND PREPARE

Explore the Concept
- Students will construct similar triangles.
- This activity leads into the study of dilations in this lesson, Example 1.

Materials
Each student will need:
- graph paper
- compass
- straightedge
- ruler

Recommended Time
Work activity: 10 min

Discuss results: 5 min

Grouping
Students can work individually or in groups of two. If students work in groups, they can share their results.

2 TEACH

Tips for Success
Point out the importance of using a compass for step 3 of the dilation. This will help students learn how to perform a dilation independent of the units on the graph paper.

Alternative Strategy
You may want to do this activity as a demonstration with drawing software.

Key Discovery
Dilations create similar images.

3 ASSESS AND RETEACH

1. A right triangle with side lengths 5, 12, 13 is dilated with a scale factor of 3. What are the side lengths of the image? **15, 36, 39**

2. The vertices of an image with center (0, 0) has vertices that are $\frac{1}{2}$ the distance from the origin as the original vertices. What is the ratio of the side lengths between the original figure and its image? **2 : 1**

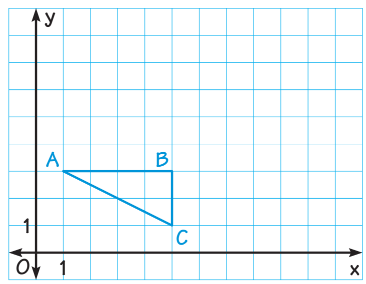
Dilations

MATERIALS • graph paper • straightedge • compass • ruler

QUESTION **How can you construct a similar figure?**

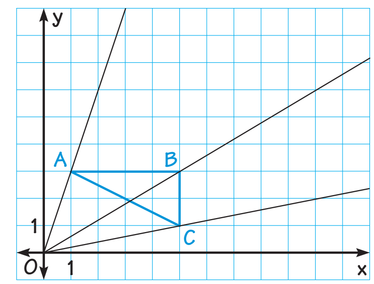
EXPLORE **Construct a similar triangle**

STEP 1



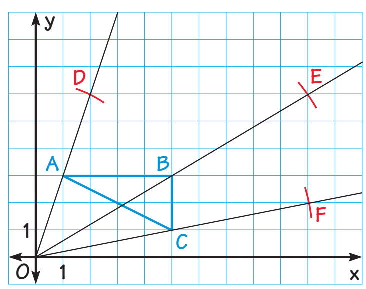
Draw a triangle Plot the points $A(1, 3)$, $B(5, 3)$, and $C(5, 1)$ in a coordinate plane. Draw $\triangle ABC$.

STEP 2



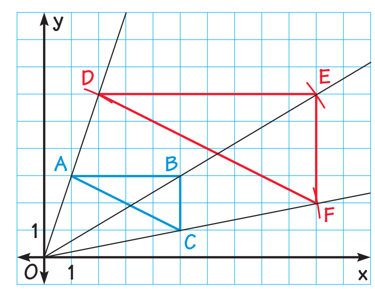
Draw rays Using the origin as an endpoint O, draw $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$.

STEP 3



Draw equal segments Use a compass to mark a point D on $\overrightarrow{OA}$ so $OA = AD$. Mark a point E on $\overrightarrow{OB}$ so $OB = BE$. Mark a point F on $\overrightarrow{OC}$ so $OC = CF$.

STEP 4



Draw the image Connect points D, E, and F to form a right triangle.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Measure $\overline{AB}$, $\overline{BC}$, $\overline{DE}$, and $\overline{EF}$. Calculate the ratios $\frac{DE}{AB}$ and $\frac{EF}{BC}$. Using this information, show that the two triangles are similar. **See margin.**

2. Repeat the steps in the Explore to construct $\triangle GHJ$ so that $3 \cdot OA = AG$, $3 \cdot OB = BH$, and $3 \cdot OC = CJ$. **See margin.**

1. $\frac{DE}{AB} = \frac{EF}{BC}$ **or** $\frac{8}{4} = \frac{4}{2}$**; since the corresponding sides are proportional and the included angles,** $\angle E$ **and** $\angle B$**, are both 90°,** $\triangle ABC \sim \triangle DEF$ **by the SAS Similarity Theorem.**

2. See Additional Answers.

6.6 Perform Similarity Transformations

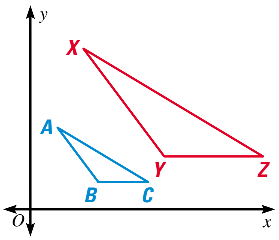
Before You performed congruence transformations.
Now You will perform dilations.
Why? So you can solve problems in art, as in Ex. 26.

Key Vocabulary
- **dilation**
- **center of dilation**
- **scale factor of a dilation**
- **reduction**
- **enlargement**
- **transformation**

A **dilation** is a transformation that stretches or shrinks a figure to create a similar figure. A dilation is a type of *similarity transformation*.

In a dilation, a figure is enlarged or reduced with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** is the ratio of a side length of the image to the corresponding side length of the original figure. In the figure shown, $\triangle XYZ$ is the image of $\triangle ABC$. The center of dilation is (0, 0) and the scale factor is $\frac{XY}{AB}$.



KEY CONCEPT *For Your Notebook*

Coordinate Notation for a Dilation

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a **reduction**. If $k > 1$, the dilation is an **enlargement**.

EXAMPLE 1 Draw a dilation with a scale factor greater than 1

READ DIAGRAMS
All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 1), *B*(4, 1), *C*(4, −1), and *D*(1, −1). Use a scale factor of 2.

Solution

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.

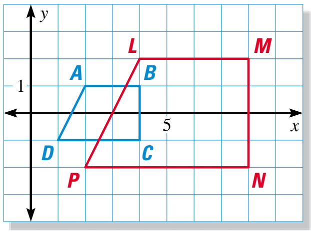
$(x, y) \rightarrow (2x, 2y)$

$A(2, 1) \rightarrow L(4, 2)$

$B(4, 1) \rightarrow M(8, 2)$

$C(4, -1) \rightarrow N(8, -2)$

$D(1, -1) \rightarrow P(2, -2)$



1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Give the coordinates of a point twice as far from the origin along a ray from (0, 0).

1. (3, 5) **(6, 10)**

2. (−2, 0) **(−4, 0)**

3. $\left(\frac{1}{2}, -\frac{1}{2}\right)$ **(1, −1)**

Give the coordinates of a point one-half as far from the origin along a ray from (0, 0).

4. (6, −4) **(3, −2)**

5. (0, −7) **(0, −3.5)**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

How do you dilate a figure in the coordinate plane? **Tell students they will learn how to answer this question by using the coordinates of the vertices of the figure.**

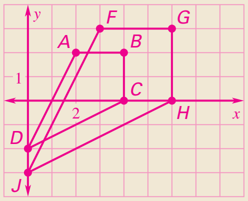
Motivating the Lesson

Place a flat object on the overhead projector and observe the image on the wall. Is the image similar to the object? Are they the same size? How could you find the scale factor? Tell students they will investigate these ideas in this lesson.

3 TEACH

Extra Example 1

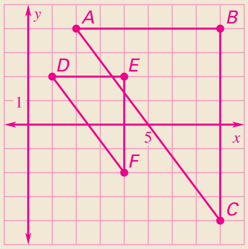
Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 2), *B*(4, 2), *C*(4, 0), *D*(0, −2). Use a scale factor of 1.5 and label the image *FGHJ*.



Extra Example 2

A triangle has vertices *A*(2, 4), *B*(8, 4), and *C*(8, −4). The image of △*ABC* after a dilation with a scale factor of $\frac{1}{2}$ is △*DEF*.

a. Sketch △*ABC* and △*DEF*.



b. Verify that △*ABC* and △*DEF* are similar. ***Sample answer:*** **∠*B* and ∠*E* are both right angles, so ∠*B* ≅ ∠*E*.** $\frac{AB}{DE} = \frac{6}{3} = 2$**, and** $\frac{BC}{EF} = \frac{8}{4} = 2$**, so the lengths of the sides that include ∠*B* and ∠*E* are proportional. Therefore △*ABC* ~ △*DEF* by the SAS Similarity Theorem.**

EXAMPLE 2 Verify that a figure is similar to its dilation

A triangle has the vertices *A*(4, −4), *B*(8, 2), and *C*(8, −4). The image of △*ABC* after a dilation with a scale factor of $\frac{1}{2}$ is △*DEF*.

a. Sketch △*ABC* and △*DEF*.

b. Verify that △*ABC* and △*DEF* are similar.

Solution

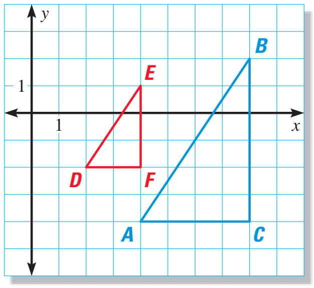
a. The scale factor is less than one, so the dilation is a reduction.

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$A(4, -4) \rightarrow D(2, -2)$

$B(8, 2) \rightarrow E(4, 1)$

$C(8, -4) \rightarrow F(4, -2)$



b. Because ∠*C* and ∠*F* are both right angles, ∠*C* ≅ ∠*F*. Show that the lengths of the sides that include ∠*C* and ∠*F* are proportional. Find the horizontal and vertical lengths from the coordinate plane.

$$\frac{AC}{DF} \stackrel{?}{=} \frac{BC}{EF} \quad \Rightarrow \quad \frac{4}{2} = \frac{6}{3} \checkmark$$

So, the lengths of the sides that include ∠*C* and ∠*F* are proportional.

▶ Therefore, △*ABC* ~ △*DEF* by the SAS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the coordinates of *L*, *M*, and *N* so that △*LMN* is a dilation of △*PQR* with a scale factor of *k*. Sketch △*PQR* and △*LMN*. **1, 2. See margin for art.**

1. *P*(−2, −1), *Q*(−1, 0), *R*(0, −1); $k = 4$ ***L*(−8, −4), *M*(−4, 0), *N*(0, −4)**

2. *P*(5, −5), *Q*(10, −5), *R*(10, 5); $k = 0.4$ ***L*(2, −2), *M*(4, −2), *N*(4, 2)**

EXAMPLE 3 Find a scale factor

PHOTO STICKERS You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of the reduction?

Solution

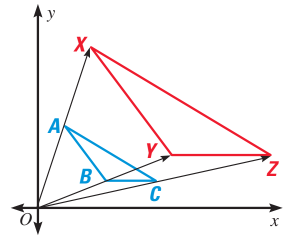
The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$. In simplest form, the scale factor is $\frac{11}{40}$.

Differentiated Instruction

Visual Learners Instruct students to make use of different colored pencils when they are drawing their triangles in **Guided Practice Exercises 1 and 2**. Have them sketch the original figure in one color and the dilation of that figure in a different color. Point out that they can determine which figure in a set of two similar figures is a dilation of the other by looking at the scale factor to find out if it is a reduction or an enlargement of the original.

See also the *Differentiated Instruction Resources* for more strategies.

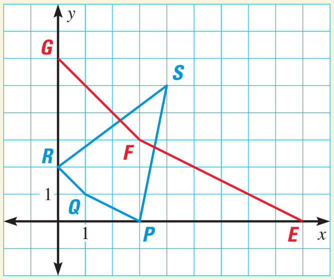
READING DIAGRAMS Generally, for a center of dilation at the origin, a point of the figure and its image lie on the same ray from the origin. However, if a point of the figure *is* the origin, its image is also the origin.



EXAMPLE 4 Standardized Test Practice

You want to create a quadrilateral *EFGH* that is similar to quadrilateral *PQRS*. What are the coordinates of *H*?

Ⓐ (12, −15)
Ⓑ (7, 8)
Ⓒ (12, 15)
Ⓓ (15, 18)



ELIMINATE CHOICES
You can eliminate choice A, because you can tell by looking at the graph that *H* is in Quadrant I. The point (12, −15) is in Quadrant IV.

Solution

Determine if *EFGH* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *E*, *F*, and *G* from *P*, *Q*, and *R*.

$(x, y) \to (kx, ky)$

$P(3, 0) \to E(9, 0)$ $\quad$ $k = 3$

$Q(1, 1) \to F(3, 3)$ $\quad$ $k = 3$

$R(0, 2) \to G(0, 6)$ $\quad$ $k = 3$

Because *k* is the same in each case, the image is a dilation with a scale factor of 3. So, you can use the scale factor to find the image *H* of point *S*.

$S(4, 5) \to H(3 \cdot 4, 3 \cdot 5) = H(12, 15)$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

CHECK Draw rays from the origin through each point and its image.

GUIDED PRACTICE for Examples 3 and 4

3. **WHAT IF?** In Example 3, what is the scale factor of the reduction if your photo is 5.5 inches by 5.5 inches? $\frac{1}{5}$

4. Suppose a figure containing the origin is dilated. *Explain* why the corresponding point in the image of the figure is also the origin.
A dilation with respect to the origin and scale factor *k* can be described as $(x, y) \to (kx, ky)$. If $(x, y) = (0, 0)$, then $(kx, ky) = (k \cdot 0, k \cdot 0) = (0, 0)$.

Extra Example 3

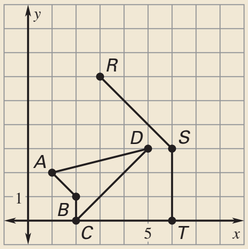
You are using a photo quality printer to enlarge a digital picture. The picture on the computer screen is 6 centimeters by 6 centimeters. The printed image is 15 centimeters by 15 centimeters. What is the scale factor of the enlargement? **5:2**

Key Question to Ask for Example 3

- If the image stickers are 1.1 inches by 2.2 inches, would the reduction be a dilation? **No, the image is not a square so it is not similar to the original.**

Extra Example 4

You want to create quadrilateral *RSTU* that is similar to quadrilateral *ABCD*. What are the coordinates of *U*? **D**



Ⓐ (15, −9) Ⓑ (8, 6)
Ⓒ (10, 6) Ⓓ (15, 9)

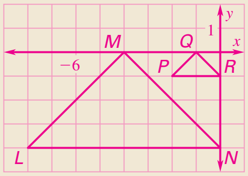
Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you dilate a figure in the coordinate plane?

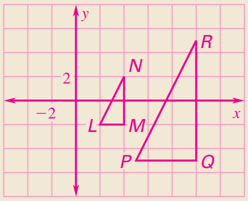
- **A dilation with respect to the origin can be described by $(x, y) \to (kx, ky)$ where *k* is the scale factor.**

Draw the figure, multiply each of the coordinates by the scale factor, and graph the new coordinates.

1.



2.



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: EP for 4.8 Exs. 36–38
Exs. 1–8, 15–18
Day 2:
Exs. 9–14, 25–30

Average:
Day 1:
Exs. 1–8, 15–18, 22
Day 2:
Exs. 9–14, 19–21, 25–31

Advanced:
Day 1:
Exs. 1, 2, 5–8, 15–18, 22–24*
Day 2:
Exs. 9–14, 19–21, 26–34*

Block:
Exs. 1–22, 25–31

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 9, 13, 25, 28
Average: 6, 10, 13, 26, 29
Advanced: 8, 12, 13, 26, 30

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

6.6 EXERCISES

HOMEWORK KEY
○ = See WORKED-OUT SOLUTIONS Exs. 5, 11, and 27
★ = STANDARDIZED TEST PRACTICE Exs. 2, 13, 21, 22, 28, 30, and 31

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: In a dilation, the image is __?__ to the original figure. **similar**

2. ★ **WRITING** *Explain* how to find the scale factor of a dilation. How do you know whether a dilation is an enlargement or a reduction? **See margin.**

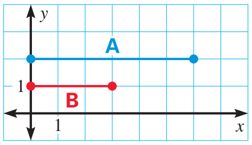
EXAMPLES 1 and 2 for Exs. 3–8

DRAWING DILATIONS **Draw a dilation of the polygon with the given vertices using the given scale factor k.** **3–8. See margin.**

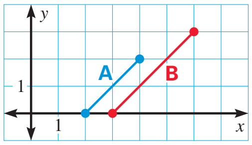
3. $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2$
4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = \frac{3}{5}$
5. $A(1, 1)$, $B(6, 1)$, $C(6, 3)$; $k = 1.5$
6. $A(2, 8)$, $B(8, 8)$, $C(16, 4)$; $k = 0.25$
7. $A(-8, 0)$, $B(0, 8)$, $C(4, 0)$, $D(0, -4)$; $k = \frac{3}{8}$
8. $A(0, 0)$, $B(0, 3)$, $C(2, 4)$, $D(2, -1)$; $k = \frac{13}{2}$

EXAMPLE 3 for Exs. 9–12

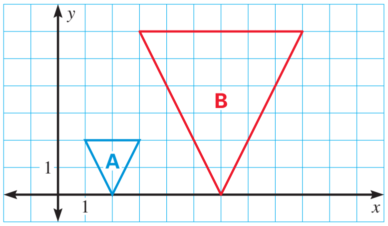
IDENTIFYING DILATIONS **Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.**

9. 

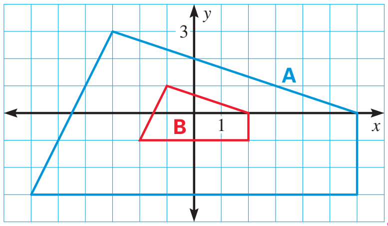
reduction; $\frac{1}{2}$

10. 

enlargement; $\frac{3}{2}$

11. 

enlargement; 3

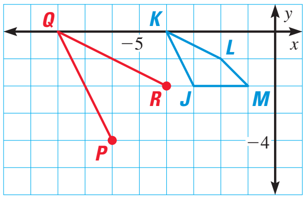
12. 

reduction; $\frac{1}{3}$

2. Find the ratio of a side length of the image to the corresponding side length of the original figure; suppose k is the scale factor, if $0 < k < 1$ the dilation is a reduction. If $k > 1$ the dilation is an enlargement.

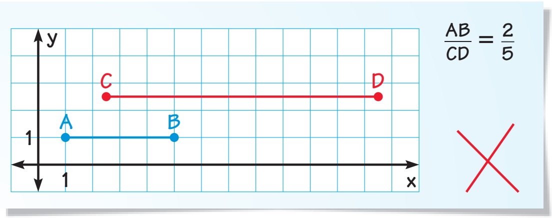
EXAMPLE 4 for Ex. 13

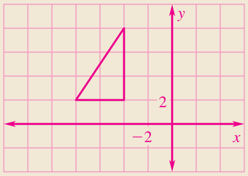
13. ★ **MULTIPLE CHOICE** You want to create a quadrilateral $PQRS$ that is similar to quadrilateral $JKLM$. What are the coordinates of S? **C**

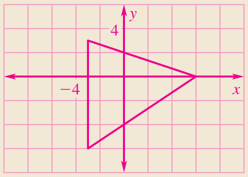


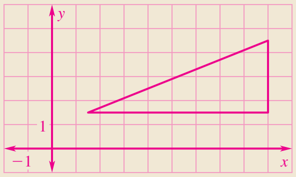
Ⓐ (2, 4) Ⓑ (4, −2)
Ⓒ (−2, −4) Ⓓ (−4, −2)

14. **ERROR ANALYSIS** A student found the scale factor of the dilation from $\overline{AB}$ to $\overline{CD}$ to be $\frac{2}{5}$. *Describe* and correct the student's error.
The scale factor should be the ratio of the image to the original rather than the ratio of the original to the image; $\frac{5}{2}$.

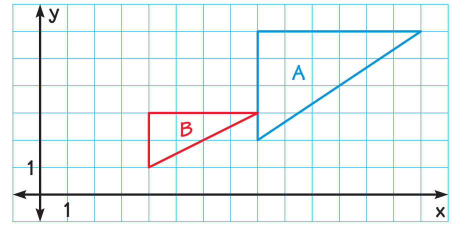


3. 

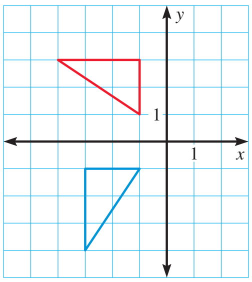
4. 

5. 

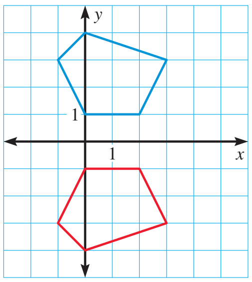
15. **ERROR ANALYSIS** A student says that the figure shown represents a dilation. What is wrong with this statement?
The figures are not similar.



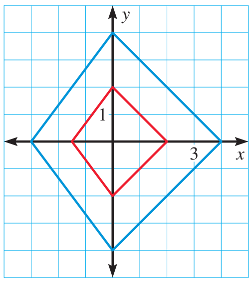
B **IDENTIFYING TRANSFORMATIONS** **Determine whether the transformation shown is a *translation*, *reflection*, *rotation*, or *dilation*.**

16. 

rotation

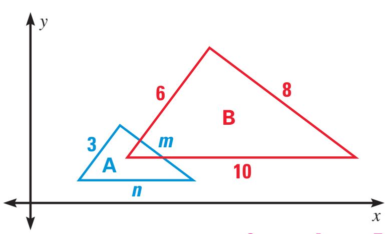
17. 

reflection

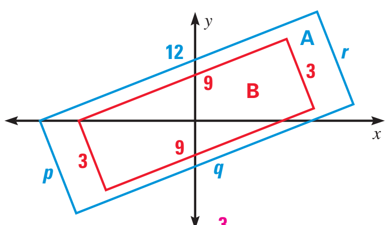
18. 

dilation

FINDING SCALE FACTORS **Find the scale factor of the dilation of Figure A to Figure B. Then give the unknown lengths of Figure A.**

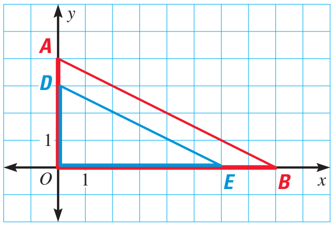
19. 

2; $m = 4$, $n = 5$

20. 

$\frac{3}{4}$; $p = 4$, $q = 12$, $r = 4$

21. ★ **MULTIPLE CHOICE** In the diagram shown, $\triangle ABO$ is a dilation of $\triangle DEO$. The length of a median of $\triangle ABO$ is what percent of the length of the corresponding median of $\triangle DEO$? **C**



Ⓐ 50% Ⓑ 75%

Ⓒ $133\frac{1}{3}\%$ Ⓓ 200%

22. ★ **SHORT RESPONSE** Suppose you dilate a figure using a scale factor of 2. Then, you dilate the image using a scale factor of $\frac{1}{2}$. *Describe* the size and shape of this new image. **The result of both dilations is the original figure.**

C **CHALLENGE** ***Describe*** **the two transformations, the first followed by the second, that combined will transform $\triangle ABC$ into $\triangle DEF$.**

23. $A(-3, 3)$, $B(-3, 1)$, $C(0, 1)$
$D(6, 6)$, $E(6, 2)$, $F(0, 2)$
Sample answer: **$(x, y) \to (-x, y) \to (2x, 2y)$**

24. $A(6, 0)$, $B(9, 6)$, $C(12, 6)$
$D(0, 3)$, $E(1, 5)$, $F(2, 5)$
Sample answer: **$(x, y) \to \left(\frac{1}{3}x, \frac{1}{3}y\right) \to (x - 2, y + 3)$**

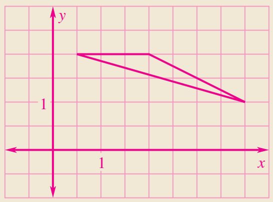
Avoiding Common Errors

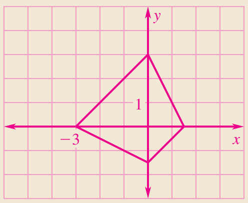
Exercises 9–13, 19, 20 Students may make errors if they confuse which figure is the image and which is the original. Encourage them to redraw the figures, using different colors for the original and the image.

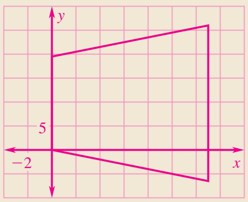
Teaching Strategy

Exercises 16–18 Review the properties of translations, reflections, rotations, and dilations for these exercises. Point out that reflections do not preserve orientation, so Exercise 17 must be a reflection.

Exercises 23–24 Encourage students to draw the original figure and the image before finding the transformations that fit these exercises. Tell them to guess and check the coordinates of the middle figure between the original and image.

6. 

7. 

8. 

PROBLEM SOLVING

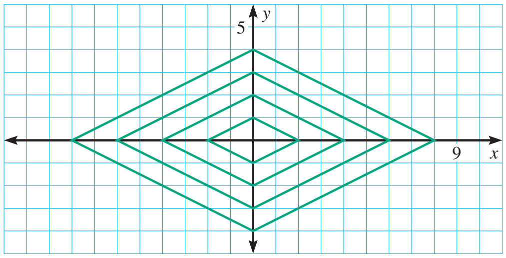
EXAMPLE 3 A
Exs. 25–27

25. BILLBOARD ADVERTISEMENT A billboard advertising agency requires each advertisement to be drawn so that it fits in a 12-inch by 6-inch rectangle. The agency uses a scale factor of 24 to enlarge the advertisement to create the billboard. What are the dimensions of a billboard, in feet? **24 ft by 12 ft**

26. POTTERY Your pottery is used on a poster for a student art show. You want to make postcards using the same image. On the poster, the image is 8 inches in width and 6 inches in height. If the image on the postcard can be 5 inches wide, what scale should you use for the image on the postcard? $\frac{5}{8}$

27. SHADOWS You and your friend are walking at night. You point a flashlight at your friend, and your friend's shadow is cast on the building behind him. The shadow is an enlargement, and is 15 feet tall. Your friend is 6 feet tall. What is the scale factor of the enlargement? $\frac{5}{2}$

28. ★ OPEN-ENDED MATH *Describe* how you can use dilations to create the figure shown below.



Multiply the coordinates of the smallest quadrilateral by 2, 3, and 4 to create each of the larger quadrilaterals.

Animated Geometry at my.hrw.com

B

29. MULTI-STEP PROBLEM $\triangle ABC$ has vertices $A(3, -3)$, $B(3, 6)$, and $C(15, 6)$.

a. Draw a dilation of $\triangle ABC$ using a scale factor of $\frac{2}{3}$. **See margin.**

b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor? **$\frac{2}{3}$; they are the same.**

c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor? **$\frac{4}{9}$; it's the square of the scale factor.**

30. ★ EXTENDED RESPONSE Look at the coordinate notation for a dilation. Suppose the definition of dilation allowed $k < 0$.

a. *Describe* the dilation if $-1 < k < 0$. **It would be a reduction.**

b. *Describe* the dilation if $k < -1$. **It would be an enlargement.**

c. Use a rotation to describe a dilation with $k = -1$. **It would be a rotation of 180°.**

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

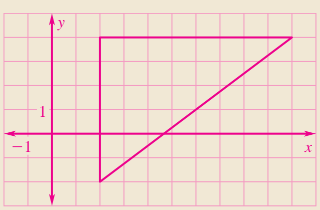
Animated Geometry
my.hrw.com

An **Animated Geometry** activity is available online for **Exercise 28**. This activity is also part of **Power Presentations**.

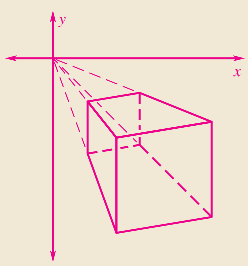
Mathematical Reasoning

Exercise 29 The ratio of corresponding segment lengths in a dilation have the same scale factor as the dilation, including the altitudes and medians of a triangle. If the scale factor is $\frac{a}{b}$, then the perimeters will have the ratio $\frac{a}{b}$, and the areas will have the ratio $\frac{a^2}{b^2}$.

29a.



31.

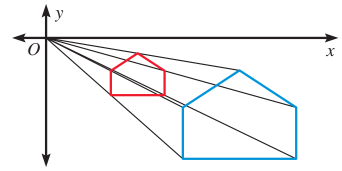


32. Let $P(a, b)$ and $Q(c, d)$ be the coordinates of the endpoints of $\overline{PQ}$ with midpoint $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$. Since $\overline{XY}$ is a dilation of $\overline{PQ}$ with scale factor k, you have $X(ka, kb)$ and $Y(kc, kd)$ with midpoint $\left(\frac{ka+kc}{2}, \frac{kb+kd}{2}\right)$. Thus, $k\left(\frac{a+c}{2}, \frac{b+d}{2}\right) = \left(\frac{ka+kc}{2}, \frac{kb+kd}{2}\right)$.

33. The slope of $\overline{PQ}$ is $\frac{d-b}{c-a}$ and the slope of $\overline{XY}$ is $\frac{kd-kb}{kc-ka} = \frac{k(d-b)}{k(c-a)} = \frac{d-b}{c-a}$. Since the slopes are the same, the lines are parallel.

31. Perspective drawings use converging lines to give the illusion that an object is three-dimensional. Since the back of the drawing is similar to the front, a dilation can be used to create this illusion with the vanishing point as the center of dilation; see margin for art.

31. ★ **SHORT RESPONSE** *Explain* how you can use dilations to make a perspective drawing with the center of dilation as a vanishing point. Draw a diagram.



C **32.** **MIDPOINTS** Let $\overline{XY}$ be a dilation of $\overline{PQ}$ with scale factor k. Show that the image of the midpoint of $\overline{PQ}$ is the midpoint of $\overline{XY}$. **See margin.**

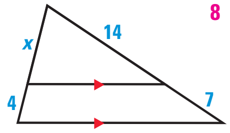
33. **REASONING** In Exercise 32, show that $\overline{XY} \parallel \overline{PQ}$. **See margin.**

34. **CHALLENGE** A rectangle has vertices $A(0, 0)$, $B(0, 6)$, $C(9, 6)$, and $D(9, 0)$. *Explain* how to dilate the rectangle to produce an image whose area is twice the area of the original rectangle. Make a conjecture about how to dilate any polygon to produce an image whose area is n times the area of the original polygon. **Use a scale factor of $\sqrt{2}$; use a scale factor of $\sqrt{n}$.**

QUIZ

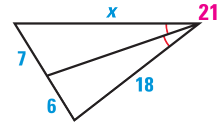
Find the value of x.

1.



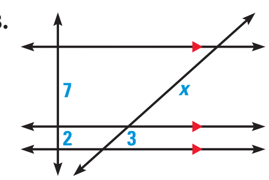
8

2.



21

3.



$\frac{21}{2}$

Draw a dilation of $\triangle ABC$ with the given vertices and scale factor k. **4, 5. See margin.**

4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = 0.4$

5. $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2.5$

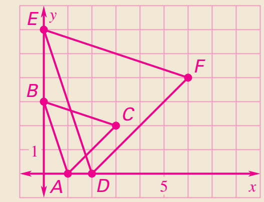
See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

5 ASSESS AND RETEACH

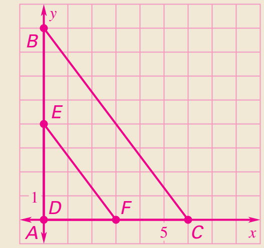
Daily Homework Quiz

Also available online

1. Draw a dilation with scale factor 2 of $\triangle ABC$ with vertices $A(1, 0)$, $B(0, 3)$, and $C(3, 2)$. Label the image $\triangle DEF$.



2. $\triangle ABC$ has vertices $A(0, 0)$, $B(0, 8)$, and $C(6, 0)$. Draw a dilation of $\triangle ABC$ using a scale factor of $\frac{1}{2}$. Label the image $\triangle DEF$.



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

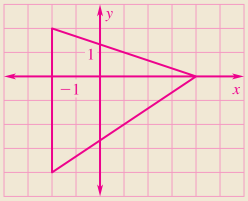
Challenge

Additional challenge is available in the Chapter Resource Book.

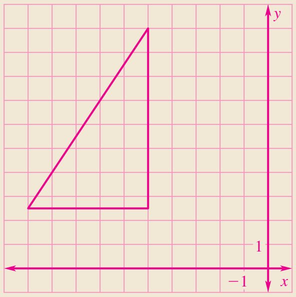
Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

4.



5.



1 PLAN AND PREPARE

Warm-Up Exercises

Find the rise and the run from point A to point B in the coordinate plane.

1. $A(-4, 3)$, $B(2, -5)$
   rise: -8; run: 6
2. $A(2, -1)$, $B(-3, -4)$
   rise: -3; run: -5
3. $A(0, -4)$, $B(1, 6)$
   rise: 10; run: 1

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How do you find the point that partitions a directed line segment in a given ratio? **Tell students they will learn how to answer this question by finding the slope of the segment and then using the given ratio.**

3 TEACH

Extra Example 1

Find the coordinates of a point P along the directed line segment AB with endpoints $A(2, 4)$ and $B(9, 6)$ so that the ratio of AP to PB is 2 to 3. **$P(4.8, 4.8)$**

Alternative Strategy

Students may want to think of percents when finding coordinates of points that partition segments. In Example 1, for instance, they may think of the point that is $\frac{3}{5}$ of the way from A to B as being "60% of the way" from A to B. Therefore, they need to add 60% of the run and 60% of the rise to the coordinates of point A in order to obtain the coordinates of point P.

Extension Partition Segments

GOAL Find the point that partitions a directed line segment in a given ratio.

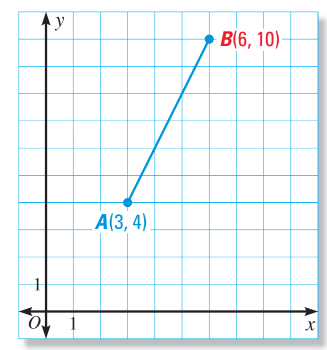
Recall that the *slope* of a nonvertical line is the ratio of *rise* (the vertical change) to *run* (the horizontal change) between any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

A *directed line segment AB* is a segment that represents moving from point A to point B. The following example shows how to use slope to find a point at a specific location on a directed line segment.

EXAMPLE 1 Find a point along a directed line segment

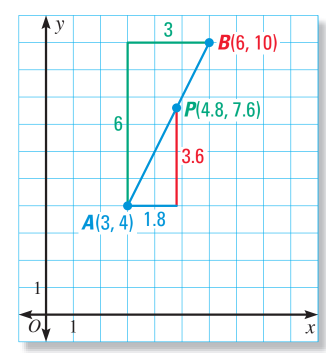
Find the coordinates of point P along the directed line segment AB so that the ratio of AP to PB is 3 to 2.



Solution

In order to divide the segment in the ratio 3 to 2, think of dividing, or *partitioning*, the segment into 3 + 2, or 5 congruent pieces.

Point P is the point that is $\frac{3}{5}$ of the way *from* point A *to* point B.



The diagram shows the rise and run from point A to point B.

$$\text{slope of } \overline{AB} = \frac{10-4}{6-3} = \frac{6}{3} = \frac{\text{rise}}{\text{run}}$$

To find the coordinates of point P, add $\frac{3}{5}$ of the run to the x-coordinate of A, and add $\frac{3}{5}$ of the rise to the y-coordinate of A.

AVOID ERRORS
Do not simplify the slope to lowest terms.

run: $\frac{3}{5}$ of $3 = 1.8$

rise: $\frac{3}{5}$ of $6 = 3.6$

▸ So, the coordinates of P are $(3 + 1.8, 4 + 3.6) = (4.8, 7.6)$. The ratio of AP to PB is 3 to 2.

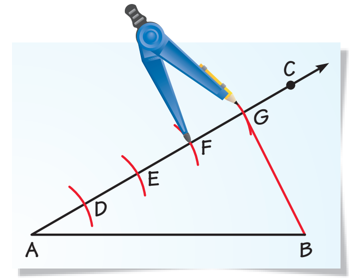
EXAMPLE 2 Construct a point along a directed line segment

Construct the point L on $\overline{AB}$ so that the ratio of AL to LB is 3 to 1.

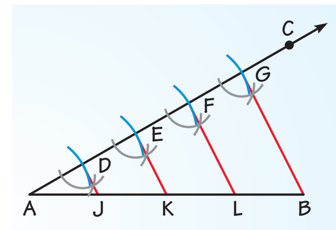
Solution

STEP 1 **Draw** $\overline{AB}$ of any length. Choose any point C not on $\overleftrightarrow{AB}$. Draw $\overrightarrow{AC}$.

STEP 2 **Place** the point of a compass at A and make an arc of any radius intersecting $\overrightarrow{AC}$ at D. Using the same compass setting, make three more arcs on $\overrightarrow{AC}$ as shown. Label the points of intersection E, F, and G, and note that $AD = DE = EF = FG$.



STEP 3 **Draw** $\overline{GB}$. Use the copy an angle construction to copy $\angle AGB$ at D, E, and F. The new sides are all parallel, and they intersect $\overline{AB}$ at J, K, and L, dividing $\overline{AB}$ equally, so that $AJ = JK = KL = LB$.



▶ Point L divides directed line segment AB in the ratio 3 to 1.

PRACTICE

EXAMPLE 1 for Exs. 1–4

PARTITIONING Find the coordinates of point P along the directed line segment AB so that AP to PB is the given ratio.

1. $A(1, 3)$, $B(8, 4)$; 4 to 1 **$P(6.6, 3.8)$**
2. $A(-2, 1)$, $B(4, 5)$; 3 to 7 **$P(-0.2, 2.2)$**
3. $A(8, 0)$, $B(3, -2)$; 1 to 4 **$P(7, -0.4)$**
4. $A(-2, -4)$, $B(6, 1)$; 3 to 2 **$P(2.8, -1)$**

EXAMPLE 2 for Exs. 5–8

CONSTRUCTION Draw a segment with the given length. Construct the point that divides the segment in the given ratio. **5–8. Check students' constructions.**

5. length: 3 in.; ratio: 1 to 4
6. length: 2 in.; ratio: 2 to 3
7. length: 12 cm; ratio: 1 to 3
8. length: 9 cm; ratio: 2 to 5

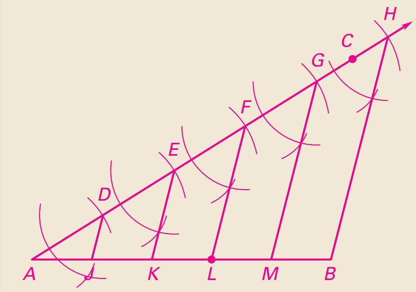
9. **REASONING** In Example 2, what theorem helps you to conclude that $AJ = JK = KL = LB$? *Explain.* **See margin.**

10. **VISUALIZATION** Suppose point P divides $\overline{XY}$ so that XP to PY is 3 to 5. Describe the point that divides $\overline{YX}$ so that YP to PX is 5 to 3. **It is point P.**

11. **WHAT IF?** Make a conjecture about how to find the coordinates of a point that lies beyond point B along $\overrightarrow{AB}$. Use an example to support your conjecture. **See margin.**

Extra Example 2

Construct the point L on a directed line segment AB so that the ratio of AL to LB is 3 to 2.



Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you find the point that partitions a directed line segment in a given ratio?

- **A directed line segment AB is a segment that represents moving from point A to point B.**

First find the rise and the run of the given segment. Then multiply the run by the final ratio of the partitioned segments and add that value to the x-coordinate of the first point. Finally, multiply the rise by the final ratio of the partitioned segments and add that value to the y-coordinate of the first point.

4 PRACTICE AND APPLY

Avoiding Common Errors

Exercises 5–8 Some students may construct the wrong number of congruent segments for these exercises. Remind them to add the two numbers given in the ratio to determine the total number of congruent segments that need to be constructed.

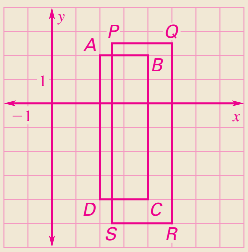
9. *Sample answer:* If parallel lines intersect two transversals, they divide the transversals proportionally. Since $AD = DE = EF = FG$, any two segments have a ratio of 1. Therefore any two segments of $\overline{AB}$ have a ratio of 1, which means $AJ = JK = KL = LB$.

11. *Sample answer:* To find a point that lies beyond point B, use a fraction that is greater than 1 along with the rise and run from A to B to find the required coordinates.

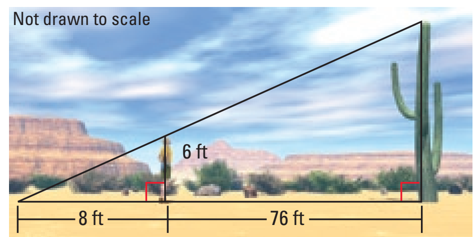
MIXED REVIEW *of Problem Solving*

2. Sometimes. ***Sample answer:*** **One possibility is for line ℓ_2 to be vertical with ℓ_1 and ℓ_3 being non-vertical lines so that the ratios of lengths $2x : x$ and $2y : y$ are achieved. In this case, ℓ_1, ℓ_2, and ℓ_3 are not parallel. It is also possible for lines ℓ_1, ℓ_2, and ℓ_3 to be parallel (as they appear in the diagram) with the line labeled with $2x$ and $2y$ drawn at enough of an angle to the three parallel lines so that the ratios of lengths $2x : x$ and $2y : y$ are achieved.**

5a.

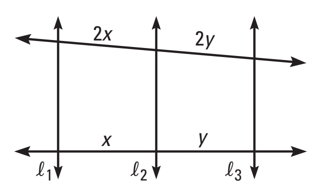


1. **SHORT RESPONSE** The Cardon cactus found in the Sonoran Desert in Mexico is the tallest type of cactus in the world. Marco stands 76 feet from the cactus so that his shadow coincides with the cactus' shadow. Marco is 6 feet tall and his shadow is 8 feet long. How tall is the Cardon cactus? *Explain.*

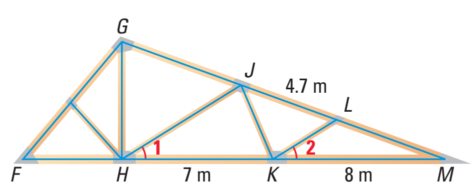


63 ft; set up the proportion $\frac{6}{8} = \frac{x}{84}$ and solve for x.

2. **SHORT RESPONSE** In the diagram, is it *always*, *sometimes*, or *never* true that $l_1 \parallel l_2 \parallel l_3$? *Explain.* **See margin.**



3. **GRIDDED ANSWER** In the diagram of the roof truss, $HK = 7$ meters, $KM = 8$ meters, $JL = 4.7$ meters, and $\angle 1 \cong \angle 2$. Find LM to the nearest tenth of a meter. **5.4 m**



4. **GRIDDED ANSWER** You are designing a catalog for a greeting card company. The catalog features a $2\frac{4}{5}$ inch by 2 inch photograph of each card. The actual dimensions of a greeting card are 7 inches by 5 inches. What is the scale factor of the reduction? **0.4**

5. **MULTI-STEP PROBLEM** Rectangle $ABCD$ has vertices $A(2, 2)$, $B(4, 2)$, $C(4, -4)$, and $D(2, -4)$.

   a. Draw rectangle $ABCD$. Then draw a dilation of rectangle $ABCD$ using a scale factor of $\frac{5}{4}$. Label the image $PQRS$. **See margin.**

   b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor? **$\frac{5}{4}$; they are the same.**

   c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor? **$\frac{25}{16}$; it's the square of the scale factor.**

6 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1

Using Ratios and Proportions to Solve Geometry Problems

You can use properties of proportions to solve a variety of algebraic and geometric problems.

For example, in the diagram above, suppose you know that $\frac{AB}{BC} = \frac{ED}{DC}$. Then you can write any of the following relationships.

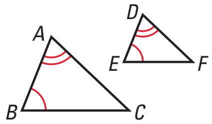
$\frac{5}{x} = \frac{6}{18}$ $\quad 5 \cdot 18 = 6x$ $\quad \frac{x}{5} = \frac{18}{6}$ $\quad \frac{5}{6} = \frac{x}{18}$ $\quad \frac{5+x}{x} = \frac{6+18}{18}$

Big Idea 2

Showing that Triangles are Similar

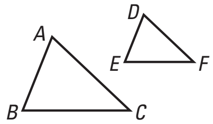
You learned three ways to prove two triangles are similar.

AA Similarity Postulate



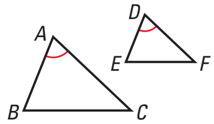
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem

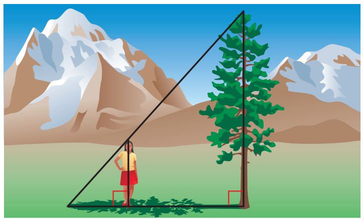


If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Big Idea 3

Using Indirect Measurement and Similarity

You can use triangle similarity theorems to apply indirect measurement in order to find lengths that would be inconvenient or impossible to measure directly.



Consider the diagram shown. Because the two triangles formed by the person and the tree are similar by the AA Similarity Postulate, you can write the following proportion to find the height of the tree.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of tree}}{\text{length of tree's shadow}}$$

You also learned about dilations, a type of similarity transformation. In a dilation, a figure is either enlarged or reduced in size.

Additional Resources

The following resources are available to help review the materials in this chapter.

Chapter Resource Book

- Chapter Review Games and Activities
- Cumulative Practice

Student Resources in Spanish

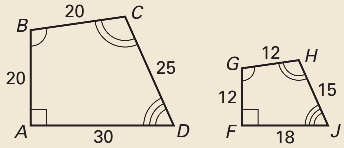
@HomeTutor

Vocabulary Practice

Vocabulary practice is available at **my.hrw.com**

Extra Example 1

In the diagram, $ABCD \sim FGHJ$. Find the scale factor.



5 : 3

6 CHAPTER REVIEW

@HomeTutor
my.hrw.com
- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see p. PT2.

- dilation
- scale factor of a dilation
- similar polygons
- scale factor of two similar polygons
- center of dilation
- reduction
- enlargement

VOCABULARY EXERCISES

Copy and complete the statement.

1. A __?__ is a transformation in which the original figure and its image are similar. **dilation**
2. If a dilation results in a figure that is smaller than the original, it is a(n) __?__. **reduction**
3. The ratio of the side lengths of two similar figures is the __?__. **scale factor**

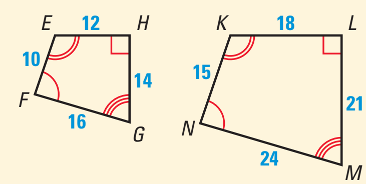
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

6.1 Use Similar Polygons

EXAMPLE

In the diagram, $EHGF \sim KLMN$. Find the scale factor.

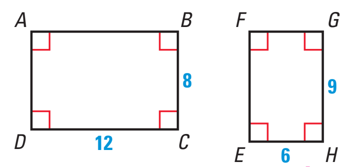


From the diagram, you can see that $\overline{EH}$ and $\overline{KL}$ correspond. So, the scale factor of $EHGF$ to $KLMN$ is $\frac{EH}{KL} = \frac{12}{18} = \frac{2}{3}$.

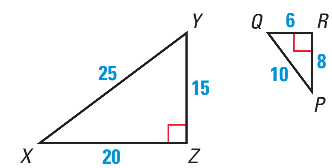
EXERCISES

EXAMPLES 2 and 4 for Exs. 4–6

In Exercises 4 and 5, determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.

4. 

similar; $ABCD \sim EFGH$, $\frac{4}{3}$

5. 

similar; $\triangle XYZ \sim \triangle PQR$, $\frac{5}{2}$

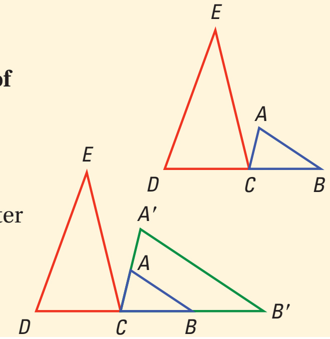
6. **POSTERS** Two similar posters have a scale factor of 4 : 5. The large poster's perimeter is 85 inches. Find the small poster's perimeter. **68 in.**

@HomeTutor
my.hrw.com
Chapter Review Practice

6.2 Relate Transformations and Similarity

EXAMPLE

***ABC* is similar to *DEC*. Describe a combination of transformations that moves *ABC* onto *DEC*.**



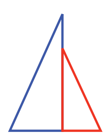
SOLUTION

A dilation with center *C* and scale factor 2 moves *ABC* onto *A'B'C*. Then a rotation of *A'B'C* with center *C* moves *A'B'C* onto *DEC*.

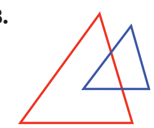
EXAMPLE 1 and 2 for Exs. 7–8

EXERCISES

The blue triangle is similar to the red triangle. Describe a combination of transformations that moves the blue triangle onto the red triangle.

7. 

***Sample answer:* dilation with scale factor $\frac{2}{3}$ followed by a reflection through a vertical line**

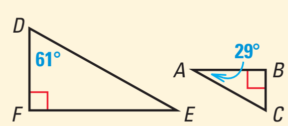
8. 

***Sample answer:* translation down and left followed by a dilation with scale factor 2**

6.3 Prove Triangles Similar by AA

EXAMPLE

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

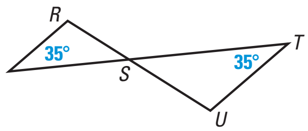


Because they are right angles, $\angle F \cong \angle B$. By the Triangle Sum Theorem, $61° + 90° + m\angle E = 180°$, so $m\angle E = 29°$ and $\angle E \cong \angle A$. Then, two angles of $\triangle DFE$ are congruent to two angles of $\triangle CBA$. So, $\triangle DFE \sim \triangle CBA$.

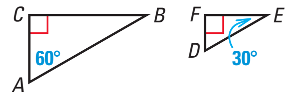
EXAMPLES 2 and 3 for Exs. 9–11

EXERCISES

Use the AA Similarity Postulate to show that the triangles are similar.

9. 

9. $\angle RSQ \cong \angle UST$ **by the Vertical Angles Theorem and it was given that** $\angle Q \cong \angle T$ **making** $\triangle SQR \sim \triangle STU$ **using the AA Similarity Postulate.**

10. 

11. CELL TOWER A cellular telephone tower casts a shadow that is 72 feet long, while a tree nearby that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower? **324 ft**

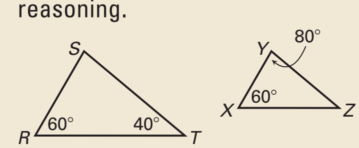
Extra Example 2

Use the art from Exercise 5 in Lesson 6.1. Describe a combination of transformations that moves *XYZ* onto *PQR*.

Translate *XYZ* so that *X* lands on *P*. Rotate *XYZ* so that $\overline{XZ}$ lands on $\overline{PR}$. Dilate *XYZ* so that *Z* lands on *R*. Then *Y* lands on *Q*.

Extra Example 3

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

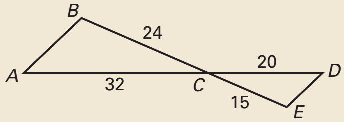


$\angle R \cong \angle X$. By the Triangle Sum Theorem, $40° + 60° + m\angle S = 180°$, so $m\angle S = 80°$, and $\angle S \cong \angle Y$. $\triangle RST \sim \triangle XYZ$ by the AA Similarity Postulate.

6 CHAPTER REVIEW

Extra Example 4

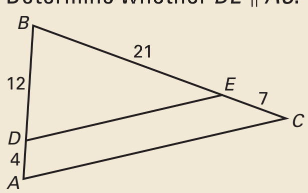
Show that the triangles are similar.



$\frac{AC}{DC} = \frac{32}{20} = \frac{8}{5}$ and $\frac{BC}{EC} = \frac{24}{15} = \frac{8}{5}$, so the sides are proportional. The included angles for these sides are vertical angles, so $\angle ACB \cong \angle DCE$. $\triangle ABC \sim \triangle DEC$ by the SAS Similarity Theorem.

Extra Example 5

Determine whether $\overline{DE} \parallel \overline{AC}$.



$\frac{BD}{DA} = \frac{12}{4} = \frac{3}{1}$, $\frac{BE}{EC} = \frac{21}{7} = \frac{3}{1}$. So $\frac{BD}{DA} = \frac{BE}{EC}$. $\overline{DE}$ is parallel to $\overline{AC}$ by Theorem 5.

12. Since $\frac{4}{8} = \frac{3.5}{7}$ and the included angle, $\angle C$, is congruent to itself, $\triangle BCD \sim \triangle ACE$ by the SAS Similarity Theorem.

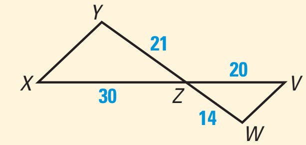
13. Since $\frac{9}{13.5} = \frac{14}{21} = \frac{10}{15}$, $\triangle QRU \sim \triangle QST$ by the SSS Similarity Theorem.

6.4 Prove Triangles Similar by SSS and SAS

10. By the Triangle Sum Theorem, $m\angle D = 60°$ so $\angle A \cong \angle D$. $\angle C \cong \angle F$ by the Right Angles Congruence Theorem. So, $\triangle ABC \sim \triangle DEF$ using the AA Similarity Postulate.

EXAMPLE

Show that the triangles are similar.



Notice that the lengths of two pairs of corresponding sides are proportional.

$$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3} \qquad \frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$$

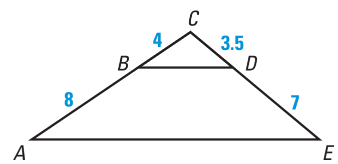
The included angles for these sides, $\angle XZY$ and $\angle VZW$, are vertical angles, so $\angle XZY \cong \angle VZW$. Then $\triangle XYZ \sim \triangle VWZ$ by the SAS Similarity Theorem.

EXAMPLE 4 for Exs. 12–13

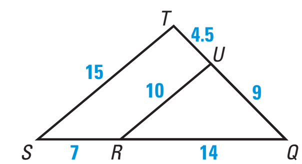
EXERCISES

Use the SSS Similarity Theorem or SAS Similarity Theorem to show that the triangles are similar. 12, 13. See margin.

12.



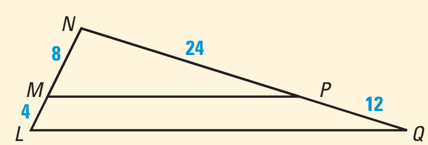
13.



6.5 Use Proportionality Theorems

EXAMPLE

Determine whether $\overline{MP} \parallel \overline{LQ}$.



Begin by finding and simplifying ratios of lengths determined by $\overline{MP}$.

$$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1} \qquad \frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1}$$

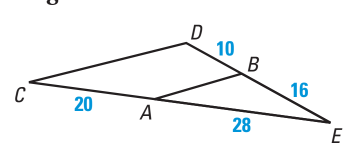
Because $\frac{NM}{ML} = \frac{NP}{PQ}$, $\overline{MP}$ is parallel to $\overline{LQ}$ by the Triangle Proportionality Converse.

EXAMPLE 2 for Exs. 14–15

EXERCISES

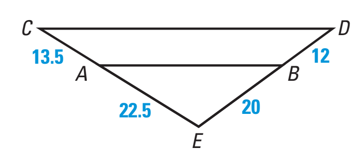
Use the given information to determine whether $\overline{AB} \parallel \overline{CD}$.

14.



not parallel

15.



parallel

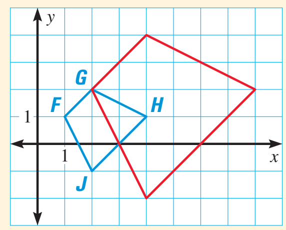
@HomeTutor
my.hrw.com
Chapter Review Practice

6.6 Perform Similarity Transformations

EXAMPLE

Draw a dilation of quadrilateral *FGHJ* with vertices *F*(1, 1), *G*(2, 2), *H*(4, 1), and *J*(2, −1). Use a scale factor of 2.

First draw *FGHJ*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.



$$(x, y) \rightarrow (2x, 2y)$$

$F(1, 1) \rightarrow (2, 2)$

$G(2, 2) \rightarrow (4, 4)$

$H(4, 1) \rightarrow (8, 2)$

$J(2, -1) \rightarrow (4, -2)$

EXERCISES

EXAMPLE 1
for Exs. 16–18

Draw a dilation of the polygon with the given vertices using the given scale factor *k*. 16–18. See margin.

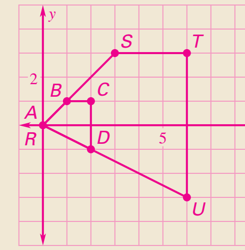
16. $T(0, 8)$, $U(6, 0)$, $V(0, 0)$; $k = \frac{3}{2}$

17. $A(6, 0)$, $B(3, 9)$, $C(0, 0)$, $D(3, 1)$; $k = 4$

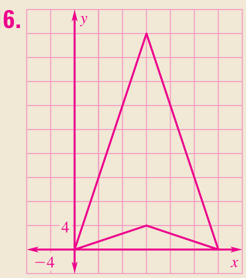
18. $P(8, 2)$, $Q(4, 0)$, $R(3, 1)$, $S(6, 4)$; $k = 0.5$

Extra Example 6

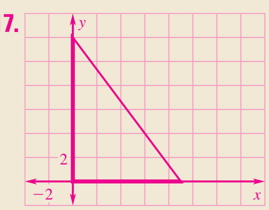
Draw a dilation of quadrilateral *ABCD* with vertices *A*(0, 0), *B*(1, 1), *C*(2, 1), and *D*(2, −1). Label the image *RSTU*. Use a scale factor of 3.



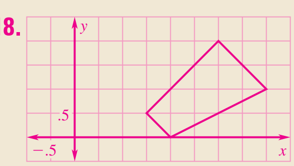
16.



17.



18.



Additional Resources

Assessment Book
- Chapter Test, Levels A, B, C
- Standardized Chapter Test
- SAT/ACT Chapter Test
- Alternative Assessment

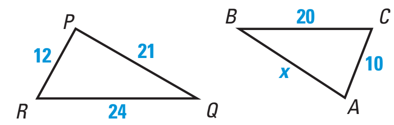
ExamView™ Assessment Suite

Chapter Test

Easily-readable reduced copies of Chapter Test B, the Standardized Chapter Test, and the Alternative Assessment from the Assessment Book can be found at the beginning of this chapter.

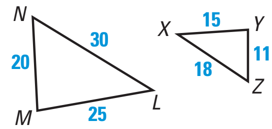
6 CHAPTER TEST

In Exercises 1–3, use the diagram where $\triangle PQR \sim \triangle ABC$.

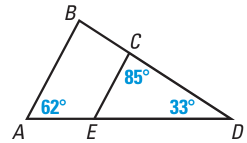


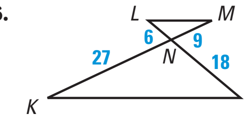
1. List all pairs of congruent angles. $\angle P$ and $\angle A$, $\angle Q$ and $\angle B$, $\angle R$ and $\angle C$
2. Write the ratios of the corresponding sides in a statement of proportionality. $\frac{x}{21} = \frac{20}{24} = \frac{10}{12}$
3. Find the value of x. 17.5

Determine whether the triangles are similar. If so, write a similarity statement and the postulate or theorem that justifies your answer.

4. 

not similar

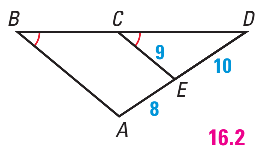
5. 

6. 

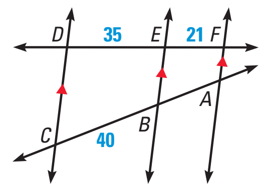
5. similar; $\triangle DEC \sim \triangle DAB$, AA Similarity Postulate

6. similar; $\triangle NKJ \sim \triangle NML$, SAS Similarity Theorem

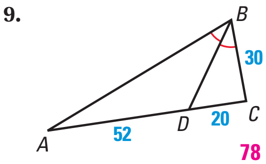
In Exercises 7–9, find the length of $\overline{AB}$.

7. 

16.2

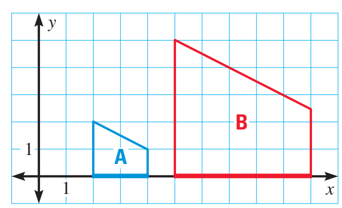
8. 

24

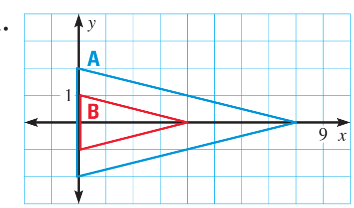
9. 

78

Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.

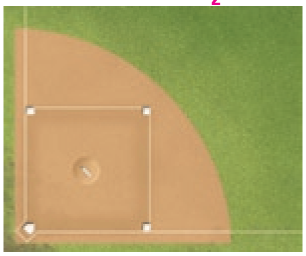
10. 

enlargement; $\frac{5}{2}$

11. 

reduction; $\frac{1}{2}$

12. **SCALE MODEL** You are making a scale model of your school's baseball diamond as part of an art project. The distance between two consecutive bases is 90 feet. If you use a scale factor of $\frac{1}{180}$ to build your model, what will be the distance around the bases on your model? 2 ft



6 ALGEBRA REVIEW

Animated Algebra my.hrw.com

SOLVE QUADRATIC EQUATIONS AND SIMPLIFY RADICALS

A radical expression is *simplified* when the radicand has no perfect square factor except 1, there is no fraction in the radicand, and there is no radical in a denominator.

EXAMPLE 1 Solve quadratic equations by finding square roots

Solve the equation $4x^2 - 3 = 109$.

$4x^2 - 3 = 109$	Write original equation.
$4x^2 = 112$	Add 3 to each side.
$x^2 = 28$	Divide each side by 4.
$x = \pm\sqrt{28}$	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, so $\sqrt{28} = \pm\sqrt{4} \cdot \sqrt{7}$.
$x = \pm 2\sqrt{7}$	Simplify.

EXAMPLE 2 Simplify quotients with radicals

Simplify the expression.

a. $\sqrt{\frac{10}{8}}$ **b.** $\sqrt{\frac{1}{5}}$

Solution

a. $\sqrt{\frac{10}{8}} = \sqrt{\frac{5}{4}}$ Simplify fraction.

$= \frac{\sqrt{5}}{\sqrt{4}}$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$= \frac{\sqrt{5}}{2}$ Simplify.

b. $\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ and $\sqrt{1} = 1$.

$= \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply numerator and denominator by $\sqrt{5}$.

$= \frac{\sqrt{5}}{5}$ Multiply fractions. $\sqrt{a} \cdot \sqrt{a} = a$.

EXERCISES

EXAMPLE 1 for Exs. 1–9

Solve the equation or write *no solution*.

1. $x^2 + 8 = 108$ ± 10
2. $2x^2 - 1 = 49$ ± 5
3. $x^2 - 9 = 8$ $\pm\sqrt{17}$
4. $5x^2 + 11 = 1$ no solution
5. $2(x^2 - 7) = 6$ $\pm\sqrt{10}$
6. $9 = 21 + 3x^2$ no solution
7. $3x^2 - 17 = 43$ $\pm 2\sqrt{5}$
8. $56 - x^2 = 20$ ± 6
9. $-3(-x^2 + 5) = 39$ $\pm 3\sqrt{2}$

EXAMPLE 2 for Exs. 10–17

Simplify the expression.

10. $\sqrt{\frac{7}{81}}$ $\frac{\sqrt{7}}{9}$
11. $\sqrt{\frac{3}{5}}$ $\frac{\sqrt{15}}{5}$
12. $\sqrt{\frac{24}{27}}$ $\frac{2\sqrt{2}}{3}$
13. $\frac{3\sqrt{7}}{\sqrt{12}}$ $\frac{\sqrt{21}}{2}$
14. $\sqrt{\frac{75}{64}}$ $\frac{5\sqrt{3}}{8}$
15. $\frac{\sqrt{2}}{\sqrt{200}}$ $\frac{1}{10}$
16. $\frac{9}{\sqrt{27}}$ $\sqrt{3}$
17. $\sqrt{\frac{21}{42}}$ $\frac{\sqrt{2}}{2}$

Extra Example 1

Solve the equation $3x^2 + 20 = 140$. $\pm 2\sqrt{10}$

Extra Example 2

Simplify the expression.

a. $\sqrt{\frac{14}{18}}$ $\frac{\sqrt{7}}{3}$

b. $\sqrt{\frac{1}{3}}$ $\frac{\sqrt{3}}{3}$

Using Rubrics

The rubric given on the pupil page is a sample of a three-level rubric. Other rubrics may contain four, five, or six levels. For more information on rubrics, see the *Differentiated Instruction Resources*.

Test-Taking Strategy

One strategy to help solve the extended response question is to make another drawing that is simplified. Separate the triangles and label each with points, dimensions, and any congruent pairs of corresponding parts. Students should note that $\angle QPR \cong \angle T$ because both are right angles and $\angle Q \cong \angle Q$ by the Reflexive Property. This lets them conclude that the triangles are similar by the AA Similarity Postulate. Then students should use the new drawing to help them write a proportion about the height of the tree.

Teaching Strategy

For part (c) of the Sample 1 solution, students may think that the student has to change his position as the position of the sun changes. Point out that it is the length of the shadow that changes, and that is reflected in the position of the point Q.

Scoring Rubric

Full Credit
- solution is complete and correct

Partial Credit
- solution is complete but has errors,

or
- solution is without error but is incomplete

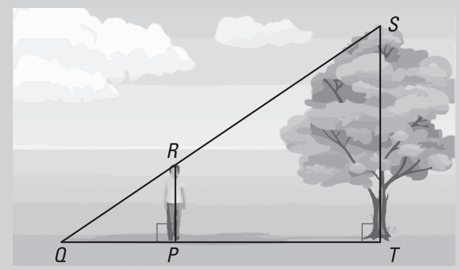
No Credit
- no solution is given,

or
- solution makes no sense

EXTENDED RESPONSE QUESTIONS

PROBLEM

To find the height of a tree, a student 63 inches in height measures the length of the tree's shadow and the length of his own shadow, as shown. The student casts a shadow 81 inches in length and the tree casts a shadow 477 inches in length.



a. Explain why $\triangle PQR \sim \triangle TQS$.

b. Find the height of the tree.

c. Suppose the sun is a little lower in the sky. Can you still use this method to measure the height of the tree? *Explain.*

Below are sample solutions to the problem. Read each solution and the comments on the left to see why the sample represents full credit, partial credit, or no credit.

SAMPLE 1: Full credit solution

The reasoning is complete.

a. Because they are both right angles, $\angle QPR \cong \angle QTS$. Also, $\angle Q \cong \angle Q$ by the Reflexive Property. So, $\triangle PQR \sim \triangle TQS$ by the AA Similarity Postulate.

The proportion and calculations are correct.

b.
$$\frac{PR}{PQ} = \frac{TS}{TQ}$$
$$\frac{63}{81} = \frac{TS}{477}$$
$$63(477) = 81 \cdot TS$$
$$371 = TS$$

In part (b), the question is answered correctly.

The height of the tree is 371 inches.

In part (c), the reasoning is complete and correct.

c. As long as the sun creates two shadows, I can use this method. Angles RPQ and T will always be right angles. The measure of $\angle Q$ will change as the sun's position changes, but the angle will still be congruent to itself. So, $\triangle PQR$ and $\triangle TQS$ will still be similar, and I can write a proportion.

SAMPLE 2: Partial credit solution

In part (a), there is no explanation of why the postulate can be applied.

In part (b), the proportion is incorrect, which leads to an incorrect solution.

In part (c), a partial explanation is given.

a. $\triangle PQR \sim \triangle TQS$ by the Angle-Angle Similarity Postulate.

b. $\frac{PR}{PQ} = \frac{TS}{TP}$

$\frac{63}{81} = \frac{TS}{396}$

$308 = TS$

The height of the tree is 308 inches.

c. As long as the sun creates two shadows, I can use this method because the triangles will always be similar.

SAMPLE 3: No credit solution

The reasoning in part (a) is incomplete.

In part (b), no work is shown.

The answer in part (c) is incorrect.

a. The triangles are similar because the lines are parallel and the angles are congruent.

b. $TS = 371$ inches

c. No. The angles in the triangle will change, so you can't write a proportion.

PRACTICE Apply the Scoring Rubric

1. A student's solution to the problem on the previous page is given below. Score the solution as *full credit, partial credit,* or *no credit.* Explain your reasoning. If you choose *partial credit* or *no credit, explain* how you would change the solution so that it earns a score of full credit.

a. $\angle QPR \cong \angle PTS$, and $\angle Q$ is in both triangles. So, $\triangle PQR \sim \triangle TQS$.

b. $\frac{PR}{PQ} = \frac{QT}{ST}$

$\frac{63}{81} = \frac{477}{x}$

$63x = 81(477)$

$x \approx 613.3$

The tree is about 613.3 inches tall.

c. The method will still work because the triangles will still be similar if the sun changes position. The right angles will stay right angles, and $\angle Q$ is in both triangles, so it does not matter if its measure changes.

Answers

1. Partial credit.

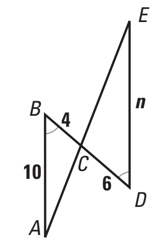
Sample answer: **In part (a) the student correctly identifies what is needed to prove the triangles similar but does not identify the theorem or postulate used.**

In part (b) the student sets up the incorrect proportion and gets the wrong answer. The proportion should be $\frac{PR}{PQ} = \frac{TS}{TQ}$.

Part (c) is correct.

EXTENDED RESPONSE

1. Use the diagram.

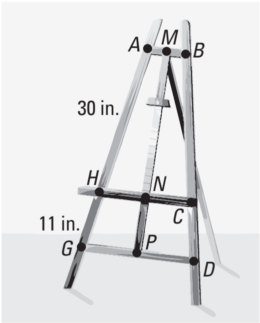


a. *Explain* how you know that $\triangle ABC \sim \triangle EDC$.

b. Find the value of n.

c. The perimeter of $\triangle ABC$ is 22. What is the perimeter of $\triangle EDC$? *Justify* your answer.

2. On the easel shown at the right, $\overline{AB} \parallel \overline{HC} \parallel \overline{GD}$, and $\overline{AG} \cong \overline{BD}$.



a. Find BD, BC, and CD. *Justify* your answer.

b. On the easel, $\overline{MP}$ is a support bar attached to $\overline{AB}$, $\overline{HC}$, and $\overline{GD}$. On this support bar, $NP = 10$ inches. Find the length of $\overline{MP}$ to the nearest inch. *Justify* your answer.

c. The support bar $\overline{MP}$ bisects $\overline{AB}$, $\overline{HC}$, and $\overline{GD}$. Does this mean that polygons $AMNH$ and $AMPG$ are similar? *Explain*.

3. A handmade rectangular rug is available in two sizes at a rug store. A small rug is 24 inches long and 16 inches wide. A large rug is 36 inches long and 24 inches wide.

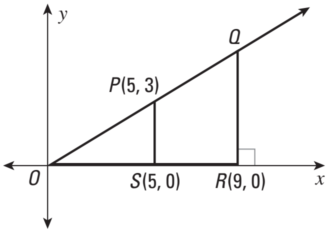
a. Are the rugs similar? If so, what is the ratio of their corresponding sides? *Explain*.

b. Find the perimeter and area of each rug. Then find the ratio of the perimeters (large rug to small rug) and the ratio of the areas (large rug to small rug).

c. It takes 250 feet of wool yarn to make 1 square foot of either rug. How many inches of yarn are used for each rug? *Explain*.

d. The price of a large rug is 1.5 times the price of a small rug. The store owner wants to change the prices for the rugs, so that the price for each rug is based on the amount of yarn used to make the rug. If the owner changes the prices, about how many times as much will the price of a large rug be than the price of a small rug? *Explain*.

4. In the diagram shown at the right, $\overleftrightarrow{OQ}$ passes through the origin.



a. Explain how you know that $\triangle OPS \sim \triangle OQR$.

b. Find the coordinates of point Q. *Justify* your answer.

c. The x-coordinate of a point on $\overleftrightarrow{OQ}$ is a. Write the y-coordinate of this point in terms of a. *Justify* your answer.

Answers

1a. $\angle ACB \cong \angle ECD$ by the Vertical Angles Congruence Theorem, $\angle B \cong \angle D$ is given, so $\triangle ABC \sim \triangle EDC$ by the AA Similarity Postulate.

1b. 15

1c. 33; the ratio of the perimeters of the two triangles is the same as the ratio of the lengths of corresponding sides in the triangles.

2a. 41 in., 30 in., 11 in.; since $\overline{AG} \cong \overline{BD}$ and $AG = 41$ inches, then $BD = 41$ inches. Theorem 6.6 along with $\overline{AG} \cong \overline{BD}$ guarantees $BC = 30$ inches and $CD = 11$ inches.

2b. 27 in.; if 3 || lines intersect 2 transversals, they divide the transversals proportionally, so $\frac{PN}{NM} = \frac{GH}{HA}$.

2c. No. *Sample answer:* The ratio $\frac{AM}{AM}$ is not equivalent to $\frac{NH}{PG}$, $\frac{MN}{MP}$, or $\frac{AH}{AG}$, so all corresponding sides are not in proportion.

3a. Yes; $\frac{2}{3}$; corresponding angles are congruent and the ratios of corresponding sides are equal.

3b. 120 in., 864 in.^{2}; 80 in., 384 in.^{2}; $\frac{3}{2}$, $\frac{9}{4}$

3c. 8,000 in., 18,000 in.; divide each area by 144, then multiply by 250 • 12.

3d. 2.25 times greater; the large rug uses 2.25 times as much yarn.

4a. AA Similarity; both triangles have $\angle O$, and $\angle PSO$ and $\angle QRS$ are right angles so they are congruent which makes the triangles similar by AA.

4b. $\left(9, \frac{27}{5}\right)$; solve the proportion $\frac{5}{9} = \frac{3}{x}$.

4c. $\frac{3}{5}a$; equation of the line is $y = \frac{3}{5}x$.

MULTIPLE CHOICE

5. If $\triangle PQR \sim \triangle STU$, which proportion is not necessarily true?

Ⓐ $\frac{PQ}{QR} = \frac{ST}{TU}$　　Ⓑ $\frac{PQ}{SU} = \frac{PR}{TU}$

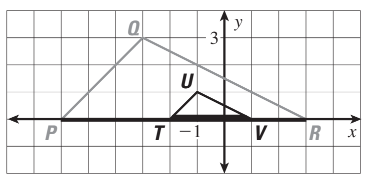
Ⓒ $\frac{PR}{SU} = \frac{QR}{TU}$　　Ⓓ $\frac{PQ}{PR} = \frac{ST}{SU}$

6. On a map, the distance between two cities is $2\frac{3}{4}$ inches. The scale on the map is 1 in.:80 mi. What is the actual distance between the two cities?

Ⓐ 160 mi　　Ⓑ 180 mi

Ⓒ 200 mi　　Ⓓ 220 mi

7. In the diagram, what is the scale factor of the dilation from $\triangle PQR$ to $\triangle TUV$?

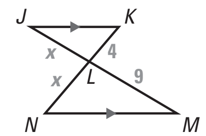


Ⓐ $\frac{1}{2}$　　Ⓑ $\frac{1}{3}$

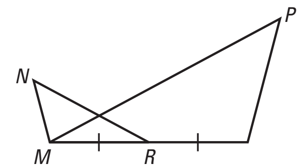
Ⓒ 2　　Ⓓ 3

GRIDDED ANSWER

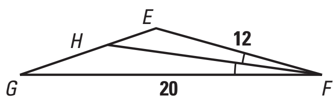
8. Find the value of x.



9. In the diagram below, $\triangle PQM \sim \triangle NMR$, and $\overline{MR} \cong \overline{QR}$. If $NR = 12$, find PM.



10. Given $GE = 10$, find HE.



11. In an acute isosceles triangle, the measures of two of the angles are in the ratio 4 : 1. Find the measure of a base angle in the triangle.

SHORT RESPONSE

12. On a school campus, the gym is 400 feet from the art studio.

a. Suppose you draw a map of the school campus using a scale of $\frac{1}{4}$ inch: 100 feet. How far will the gym be from the art studio on your map?

b. Suppose you draw a map of the school campus using a scale of $\frac{1}{2}$ inch : 100 feet. Will the distance from the gym to the art studio on this map be *greater than* or *less than* the distance on the map in part (a)? *Explain.*

13. Rectangles *ABCD* and *EFGH* are similar, and the ratio of *AB* to *EF* is 1 : 3. In each rectangle, the length is twice the width. The area of *ABCD* is 32 square inches. Find the length, width, and area of *EFGH*. *Explain.*

5. B

6. D

7. B

8. 6

9. 24

10. 3.75

11. 80°

12a. 1 in.

12b. Greater than. *Sample answer:* The distance will be 2 inches.

13. 24 in., 12 in., 288 in.^{2}; use the area and the fact that the length is twice the width to find the length and width of *ABCD* to be 8 inches and 4 inches. Use the ratio of 1 : 3 to find the length and width of *EFGH* to be 24 inches and 12 inches and the area to be 288 square inches.